A.5: Special Relativity

- https://openstax.org/books/university-physics-volume-3/pages/5-introduction
- https://youtu.be/bJMYoj4hHqU?si=H_nz648IrI92zN0C (Frames of reference)
- https://www.youtube.com/watch?v=JqwxQvq8IH0&list=PL2RRoMlng3gpCG4lWX1_BG-9D83KsKFBj&index=7 (Lorentz Transformation)
- https://www.youtube.com/watch?v=aQQXWqkGvB0&list=PL2RRoMlng3gpCG4lWX1_BG-9D83KsKFBj&index=6 (Time Dilation)
- https://www.youtube.com/watch?v=9nr9pfyD71w&list=PL2RRoMlng3gpCG4lWX1_BG-9D83KsKFBj&index=1 (A.5 with Worksheets)

Relativity *Special and General*

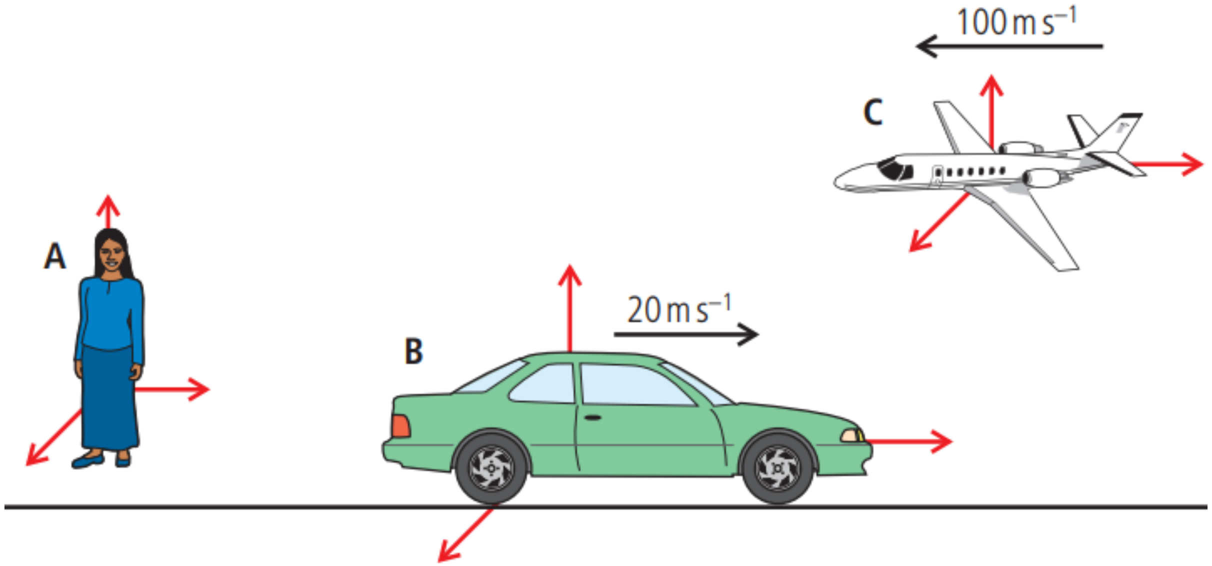
- **Relativity** is about relative motions and physics as experienced when observer and a physical phenomena do not follow the same measurement coordinates.
- **SPECIAL theory of Relativity (STR)**: About what happens to physics when relative speed of two sets of coordinates move closer to speed of light.
- **STR** (as well as GTR) are a paradigm shift. (NOS)
- It is **NOT the same** as the General theory of relativity (GTR) [Misconceptions]
- We shall NOT study the GTR.

Special Relativity

- All the mechanics you studied till now was “experiential”.
- Einstein’s Relativistic mechanics, however, is not.
- Involves very high speeds (comparable to speed of light c) unachievable for practical objects even today
- Speeds the fraction of speed of light $(c = 3\times10^8 ms^{-1})$in air/vacuum
- Special Relativity is a “paradigm shift” in understanding motion

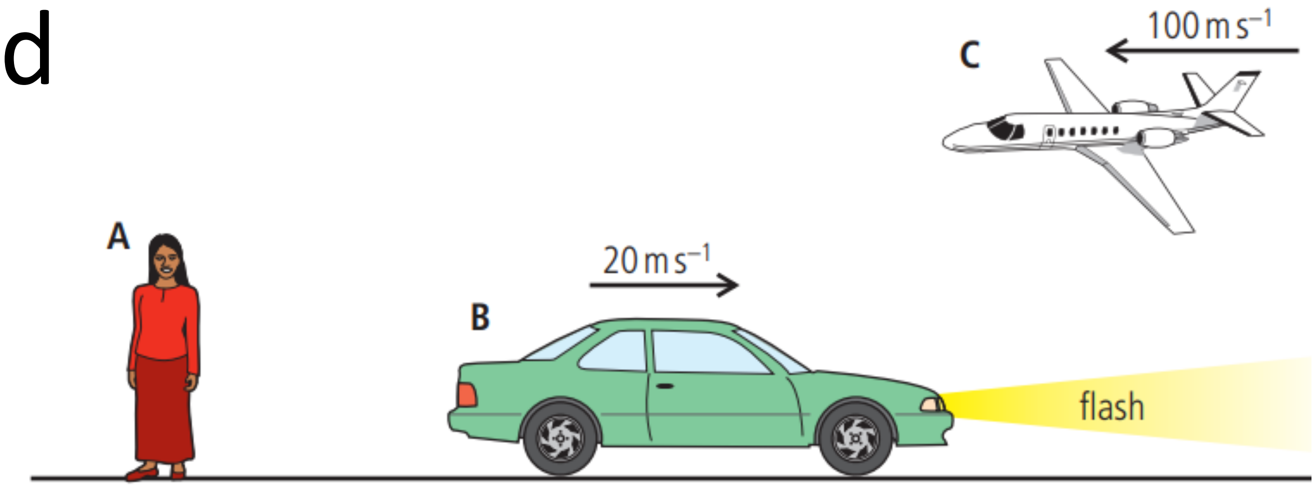
Terminology

- Event: Some change in time and space
- Observer: Someone who measures physical quantity related to an event.
- Frame of reference: System (coordinates) of measurement
- An observer can only make measurement in his/her own system



- E.g.
- **Event**: observer moving/time passing
- **Observers**: Persons at (A), in car (B), in plane (C)
- Frame of reference: Point A, Car (B), plane (C)
- An observer can only make measurement in his/her own system

Measurements and Reference Frames



- Convention: Velocities add when coming together.

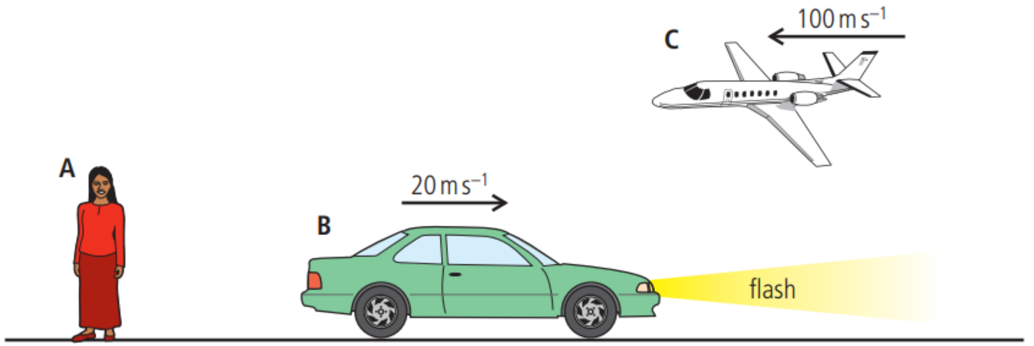
Left $\Rightarrow$ negative velocity $(v < 0)$

Right $\Rightarrow$ positive velocity $(v > 0)$

- As measured by **A**:
- Velocity of A: 0
- Velocity of car: $20\ ms^{-1}$
- Velocity of plane: $-100\ ms^{-1}$

- As Measured by **B**:
- Velocity of A: $-20\ ms^{-1}$
- Velocity of car: 0
- Velocity of plane: $-120\ ms^{-1}$

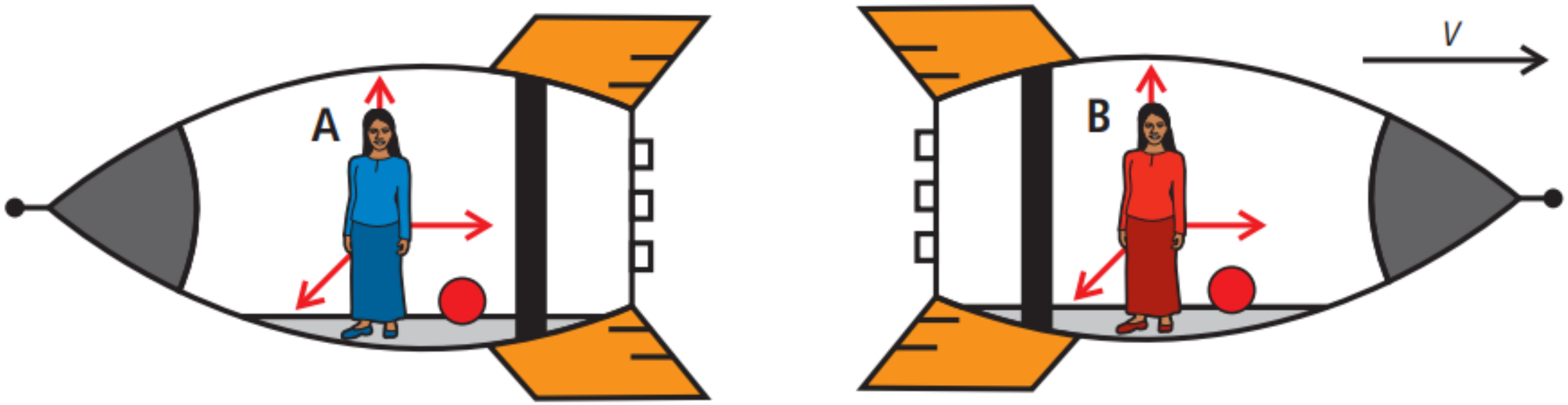
- As measured by **C**:
- Velocity of A: $100\ ms^{-1}$
- Velocity of car: $120\ ms^{-1}$
- Velocity of plane:0



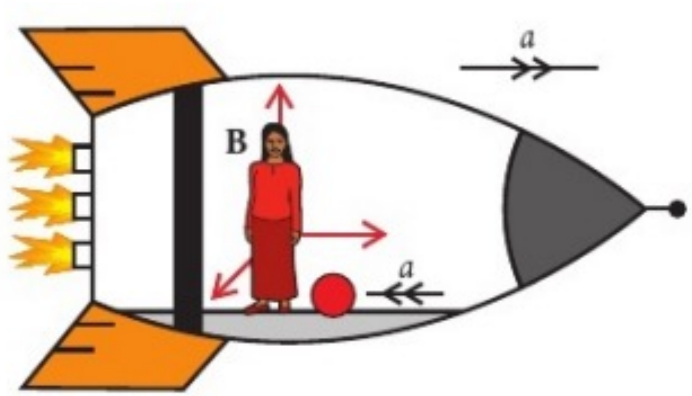
- Learning outcome:
- For measurements sake, a “frame of reference” is just a set of coordinates
- Same quantity measured from different ‘frames of reference’ will give you different measurements when they are moving with relation to each other.
- Thus, relative movement poses complications.

- *Earth is a frame of reference, and moon is another*

  Something that is stationary on earth will not be with respect to another.



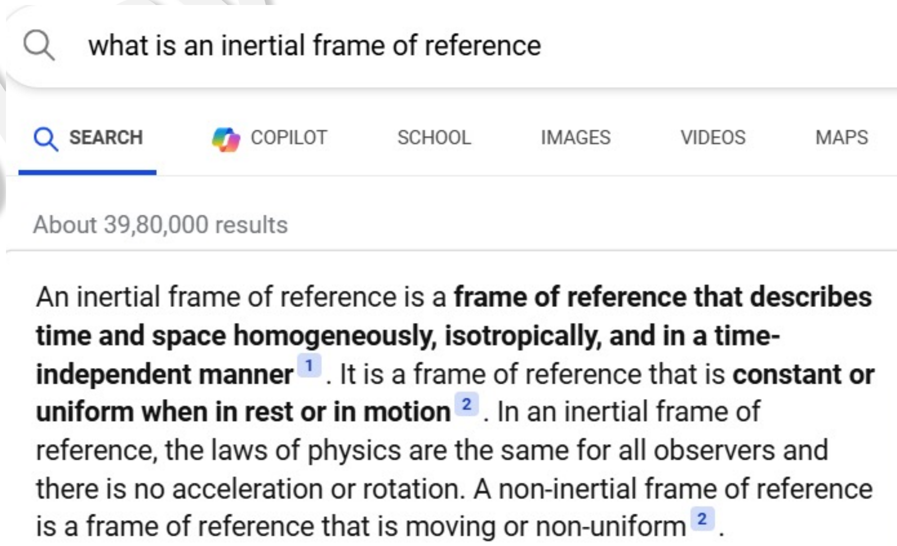
- E.g. Earth's gravity, rotation pose complications to references
- Hence, our systems will be in free space (devoid of gravity, rotation etc.)
- Case 1: Rocket B moves towards right with some '**constant**' **speed** $\boldsymbol{v}$
- **no acceleration** ⇒ **Inertial** frame of reference
- *Ball on the ground won't move unless kicked*, in ***either of the rockets***.
- Newton's laws are same individually in both frames.
  Ball on ground will not move without an external unbalanced force acting on it.



- E.g. Earth's gravity, rotation pose complications to references
- Hence, our systems will be in free space (devoid of gravity, rotation etc.)
- Case 2: Rocket B is accelerated
- Ball on ground will move without kicking ("seems" Newton's laws are broken)
- … as a reaction to force from floor. (Newton's laws are still followed)
- ***Accelerated frame is called "NON-inertial" frame of reference.***
- *We study only inertial frames (frames moving with constant speed v)*

Galilean Relativity (for *inertial frames*)

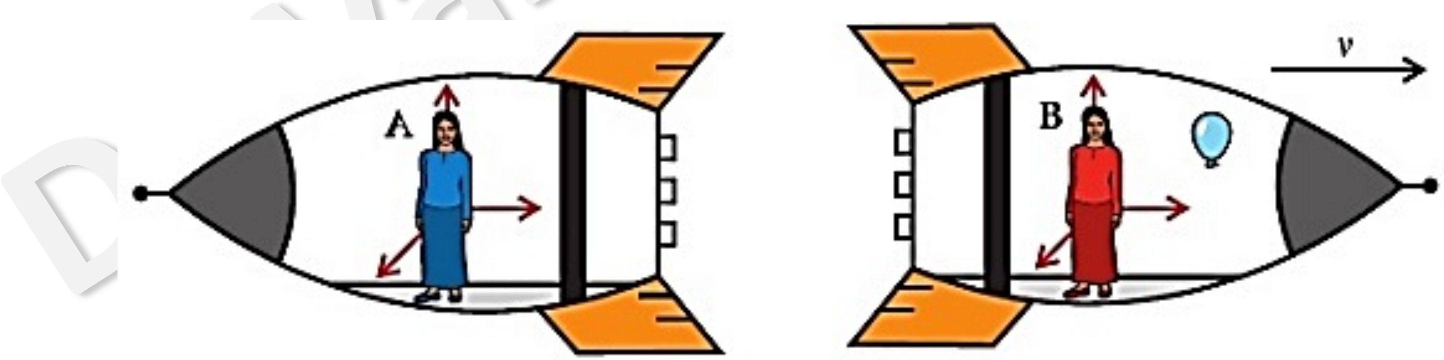
- Principle of relativity
  "Basic Laws of Physics are the same in all inertial (uniformly moving) frames of reference (individually)"



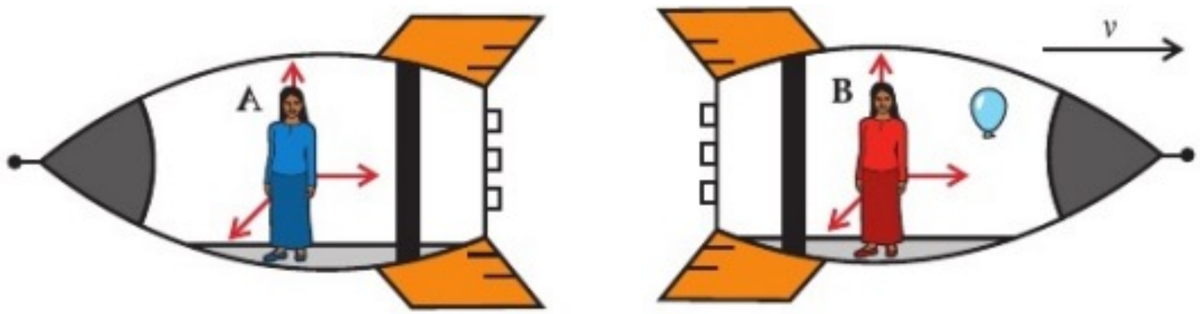
Galilean Relativity (for *inertial frames* only)

- Measurement: Position of blue balloon
- Coordinates measured by $\mathrm{A}:\ \mathrm{x}$
- Coordinates measured by $\mathrm{B}:\ \mathrm{x}'$
- v: constant speed of separation
- NOTE: ***time t is same in both frames***

A (S)	**B** (S′)	Transformation
x	x'	$x = x' + vt$
y	y'	$y = y'$
z	z'	$z = z'$
t	t'	$t = t'$



Length in Galilean Relativity

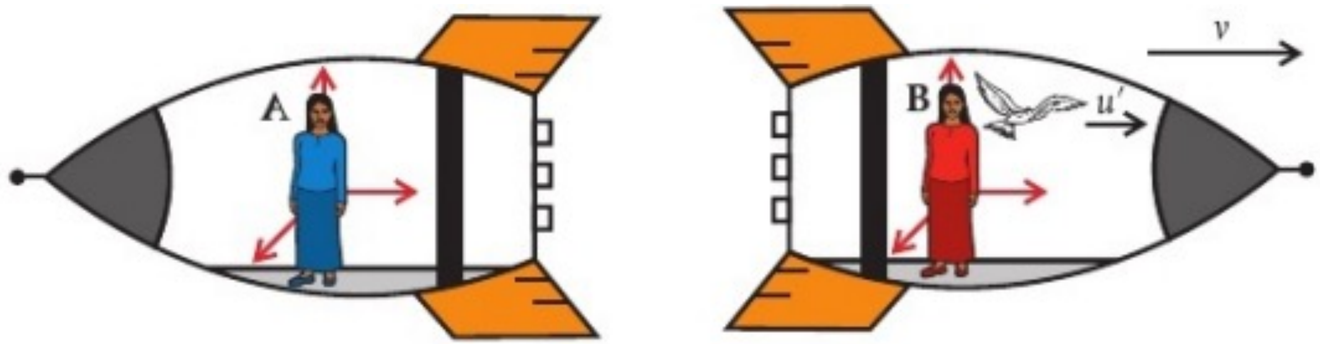


- Length is difference between positions
- $L = x_2 - x_1 = (x_2' + vt) - (x_1' + vt) = x_2' - x_1{}'$
- Length is invariant in inertial frames (Galilean relativity)

A (S)	**B** (S′)	Transformation
x	x'	$x = x' + vt$
y	y'	$y = y'$
z	z'	$z = z'$
t	t'	$t = t'$

Velocity in Galilean Relativity

- Length is difference betwee
- $L = x_2 - x_1 = (x_2' + vt) -$
- Length is invariant in inertia



A (S)	**B** (S′)	Transformation
u	u'	$u = u' + v$

- Velocity measured from A, $\boldsymbol{u} = \boldsymbol{u}' + \boldsymbol{v}$

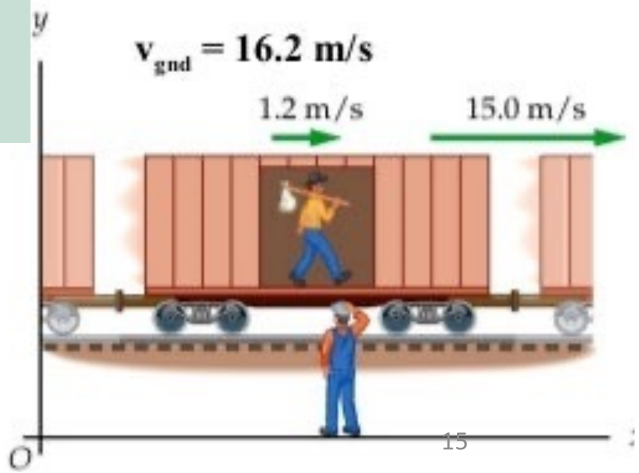
A train travels through a station at a constant velocity of 8 m s^{-1}. One observer sits on the train and another sits on the platform. As they pass each other, they start their stopwatches and take measurements of a passenger on the train who is walking in the same direction as the train.

Before starting to answer the following questions, make sure you understand what is happening; drawing a diagram will help.

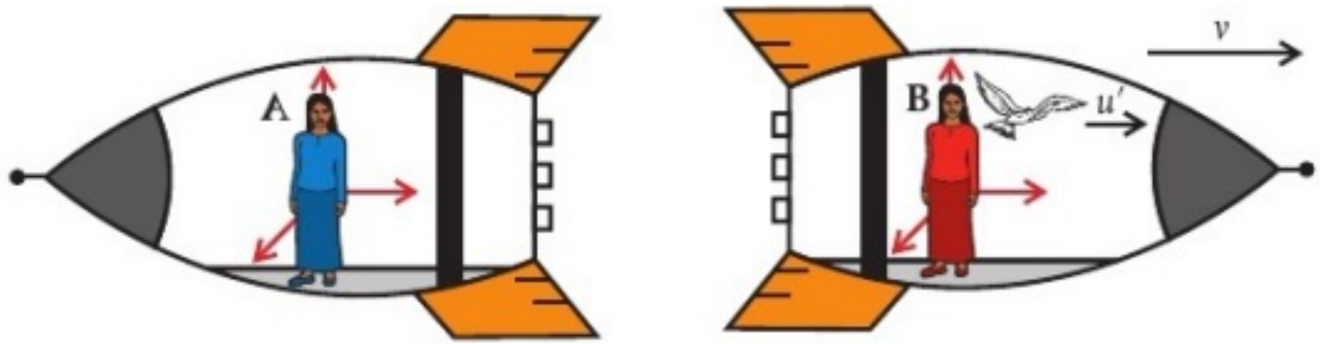
(**a**) The train observer measures the velocity of the passenger to be 0.5 m s^{-1}. What is the velocity to the platform observer?

(**b**) After 20 s, how far has the walking passenger moved according to the observer on the train?

(**c**) After 20 s, how far has the walking passenger moved according to the observer on the platform?



Acceleration in Galilean Relativity



A (S)	**B** (S′)	Transformation
u	u'	$u = u' + v$

- Length is difference betwee
- $L = x_2 - x_1 = (x_2' + vt) -$
- Length is invariant in inertia

- Velocity measured from A, $\boldsymbol{u} = \boldsymbol{u}' + \boldsymbol{v}$
- Bird accelerates from u_1' to ${u_2}'$ in time Δt
- Acceleration in B $a' = \frac{u_2' - u_1'}{\Delta t}$ as measured in A: $a = \frac{[(u_1'+v)-(u_2'+v)]}{\Delta t} = a'$
- Hence, ***acceleration is also invariant in Galilean relativity (inertial frames)***
- $\Rightarrow$ Newton's second law applies as is in inertial frame

Enter ….. Special Theory of Relativity (STR)

Convention and terminology

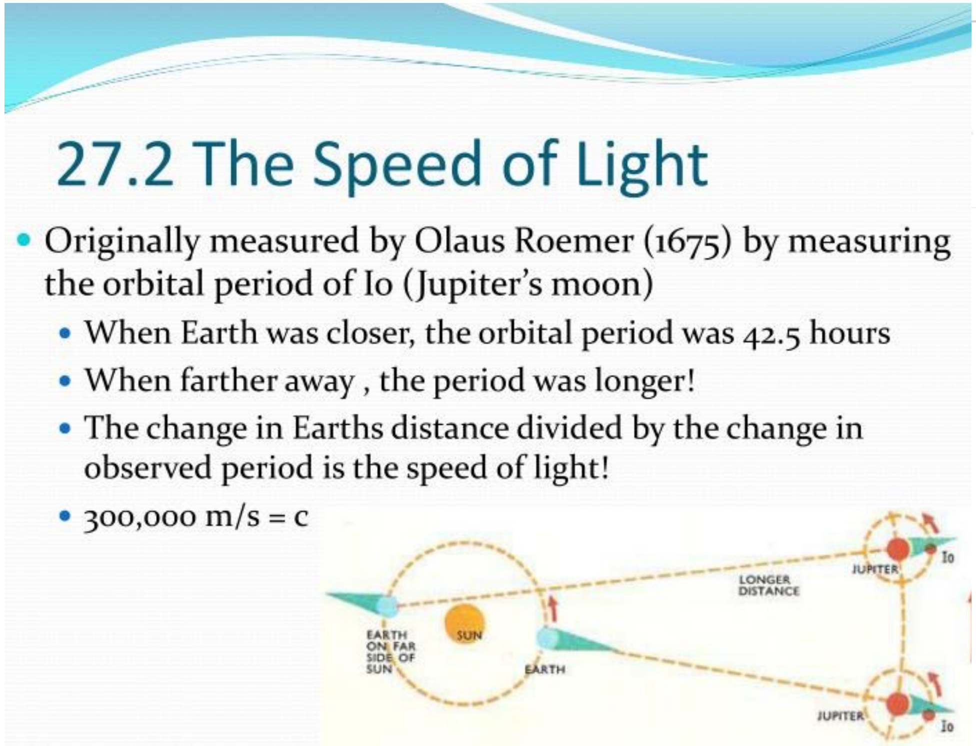
- *Measurements* made *in the same* (usually the moving) *frame of reference* where the event takes place, are '**proper**' measurements
- We shall append symbol ′ (prime) to denote these quantities

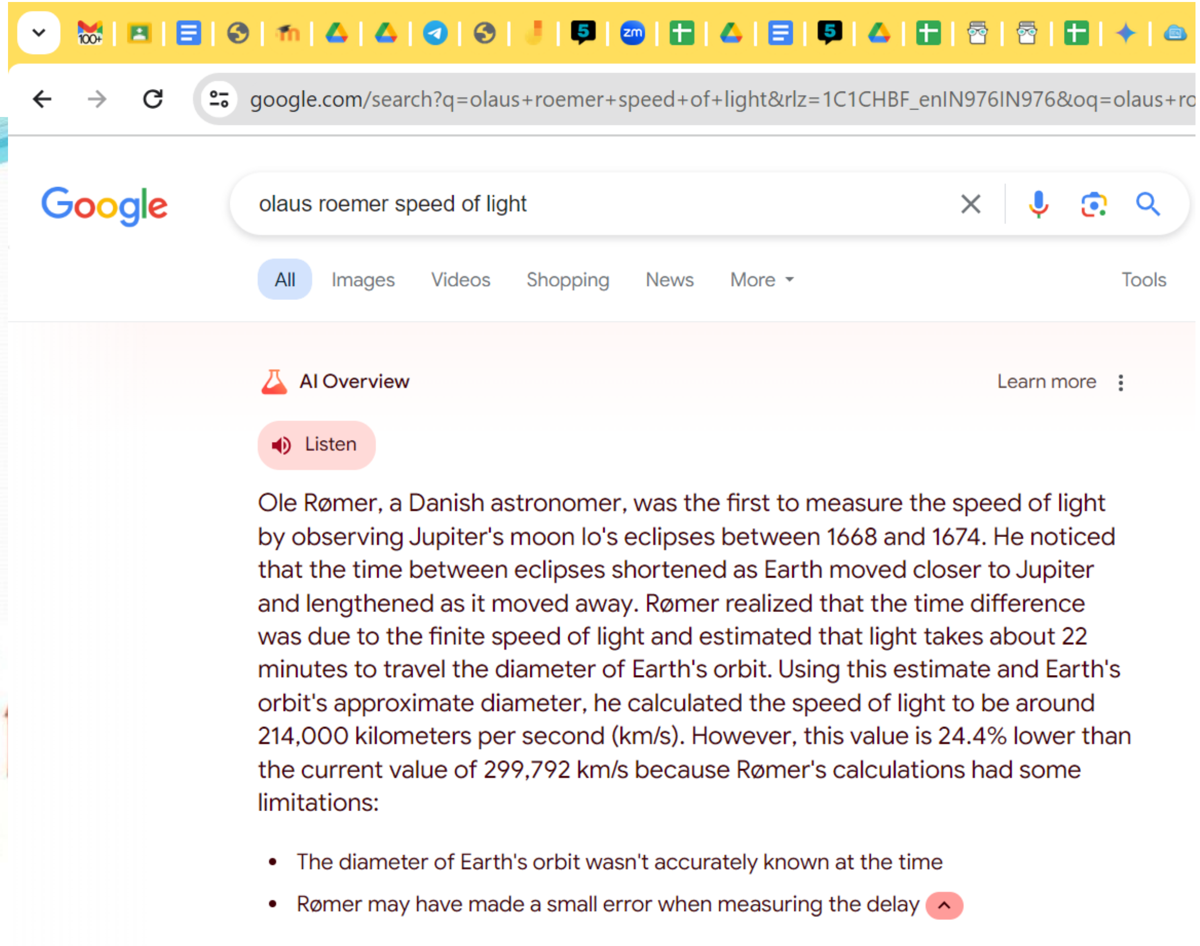
Need for the STR (Problems in the heaven!)

- Galilean relativity is logically sound and practical.
  Applies to objects we see around.
- Things like electromagnetic waves couldn't fit in these descriptions (thanks to Maxwell)
- Maxwell explained how light travelled through vacuum as wave of electric and magnetic oscillations (Electromagnetic wave)
- Theory by maxwell predicted the same speed $\boldsymbol{c} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{\varepsilon_0 \mu_0}}} \approx 3\times10^8\ ms^{-1}$
- It involves only constants of medium.
- Has not scope for inclusion of relative motion predictions at *c*.
- Speed of light measured till then (circa $\sim$1900 AD) $c \approx 3\times10^8\ ms^{-1}$

[PPT - Chapter 27: Light PowerPoint Presentation, free download - ID:1886400 (slideserve.com)](slideserve.com)
[Speed of light [3 of 4] measured by Romer | PPT (slideshare.net)](slideshare.net)

[Speed of light [3 of 4] measured by Romer | PPT (slideshare.net)]





~1675 AD

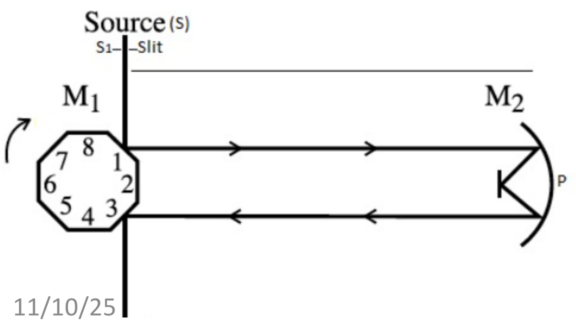
[michealsonss-method-velocity-light]

Michaelson's experiment to measure the speed of light *c*

sciencetopia.net/physics/michealsonss-method-velocity-light

Michelsons's Method is a precise method for measuring the speed of light. An octagonal mirror M_1 is mounted on the shaft of a variable speed motor. Light from a bright source *S* is focused at an angle of 45^o on one of the faces of mirror M_1 after passing through a slit S_1. The reflected light falls on a distant concave mirror M_2. In figure, M_3 is a plane mirror and with the help of this mirror M_3 placed at the center of curvature of mirror M_2 the beam of light is returned back and falls on face 3 of the octagonal mirror M_1 again at an angle of 45^o. The light reflected by this face is then collected by a telescope T and the eye of the telescope.

If the mirror M_1 is rotated, the light returning to it from the mirror M_2 will not be incident at an angle of 45^o, and hence will not enter the telescope. When the speed of rotation of the mirror M_1 is so adjusted that the face 2 of mirror occupies exactly the same position as was occupied by face 3 earlier (in $1/8^{th}$ revolution, of mirror M_1) during hte time light travels from M_1 to M_2 and back to M_1, then the image of source will reappear.



$$t = \frac{2d}{c}$$

If f is the number of revolutions per seconds of mirror M_1 and m is the number of faces of this mirror, then the angle rotated by the mirror during the time t is

$$\theta = \frac{2\pi}{m}$$

Now,

$$t = \frac{\theta}{2\pi f}$$

$$\frac{2d}{c} = \frac{\theta}{2\pi f}$$

$$c = \frac{4\pi f d}{\theta}$$

$$Since, \quad \theta = \frac{2\pi}{m}$$

$$c = 2dfm$$

https://www.sciencetopia.net/physics/michealsonss-method-velocity-light

[Speed of Light Experiment by Michelson (youtube.com)]

~1926

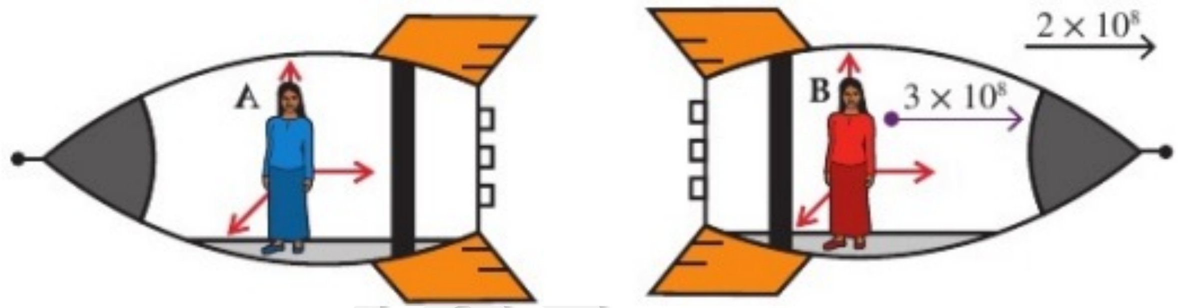
Many people measured the Speed of Light in history

<1638	Galileo, covered lanterns	inconclusive
<1667	Accademia del Cimento, covered lanterns	inconclusive
1675	Rømer and Huygens, moons of Jupiter	220 000
1729	James Bradley, aberration of light	301 000
1849	Hippolyte Fizeau, toothed wheel	315 000
1862	Léon Foucault, rotating mirror	298 000±500
1907	Rosa and Dorsey, EM constants	299 710±30
1926	**Albert A. Michelson, rotating mirror**	**299 796±4**
1950	Essen and Gordon-Smith, cavity resonator	299 792.5±3.0
1958	K.D. Froome, radio interferometry	299 792.50±0.10
1972	Evenson et al., laser interferometry	299 792.4562±0.0011
1983	17th CGPM, definition of the metre	299 792.458

[Speed of Light Experiment by Michelson (youtube.com)](https://www.youtube.com/watch?v=B_T_Xi_bd1c&t=13s)

https://www.youtube.com/watch?v=B_T_Xi_bd1c&t=13s

Speed of light in Galilean Relativity



- In Galilean relativity $u = u' + v$
- Which for speed of light in above case, means
- $c = c' + v\,;\, c = (3{\times}10^8) + (2{\times}10^8) = 5{\times}10^8\ m\,s^{-1}$
- However, this clashes with Maxwell's speed of light equations, and …
- Experiments had proven Maxwell correct.
- $c \approx 3{\times}10^8 ms^{-1}$ whether source is moving or not.
- Maxwell's momentum of light $p = E/c$ does not fit classical definition ($p = mv$)
- Einstein showed how to fit all these together in special relativity

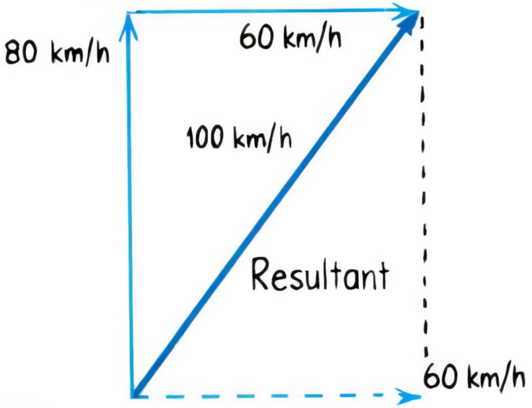
[PPT - Chapter 27: Light PowerPoint Presentation, free download - ID:1886400 (slideserve.com)]
[Speed of light [3 of 4] measured by Romer | PPT (slideshare.net)]

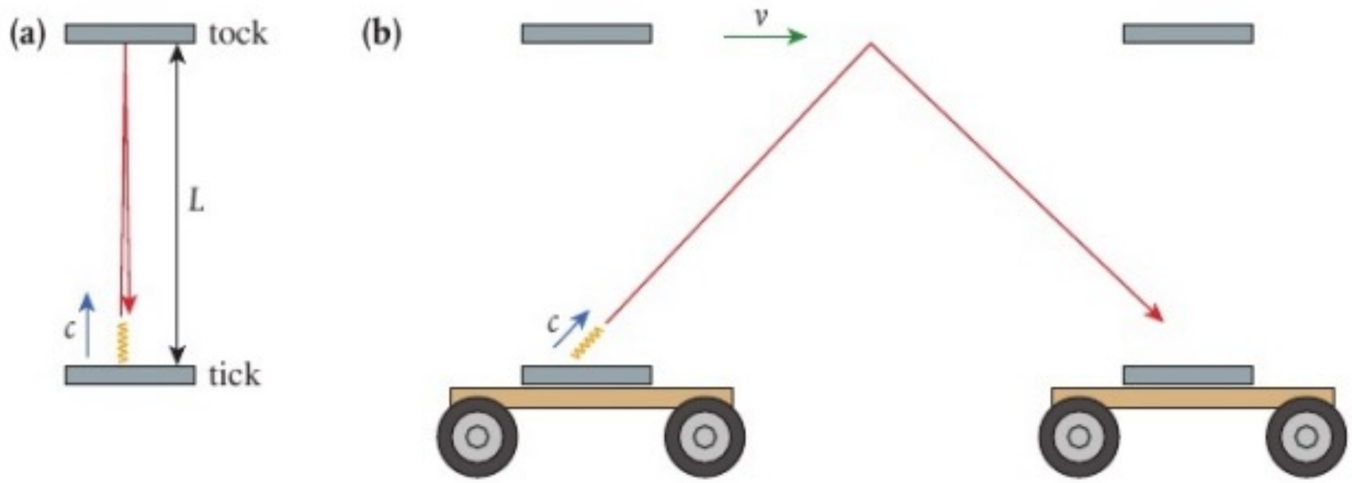
STR postulates

- *“Laws of physics are same in all inertial frames of reference”*

- *“Speed of light is the same as measured by all inertial frame observers”*

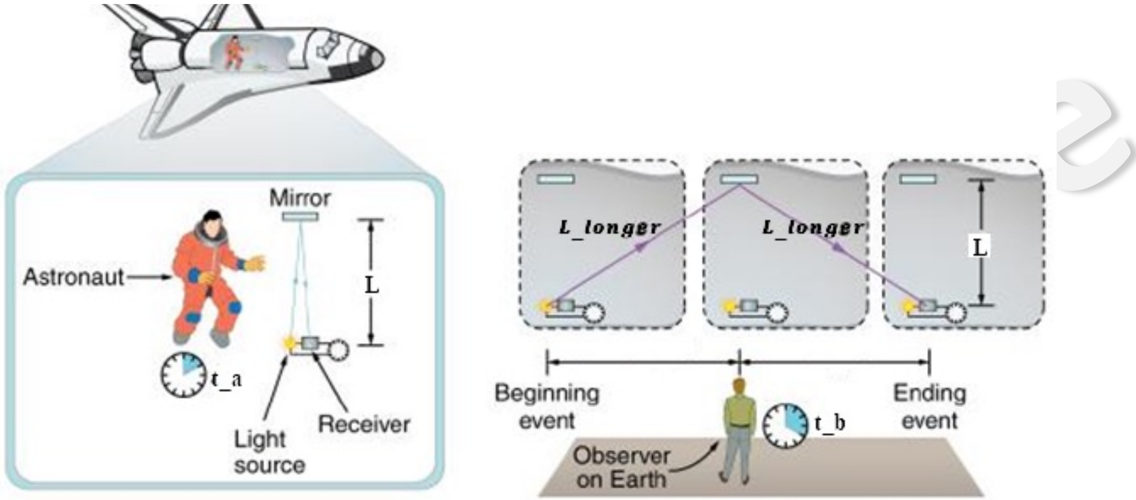
Galilean Case



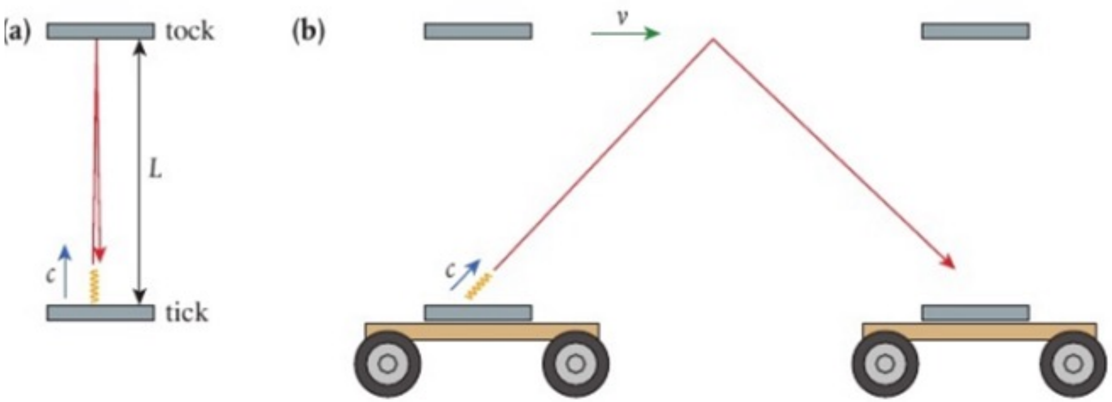


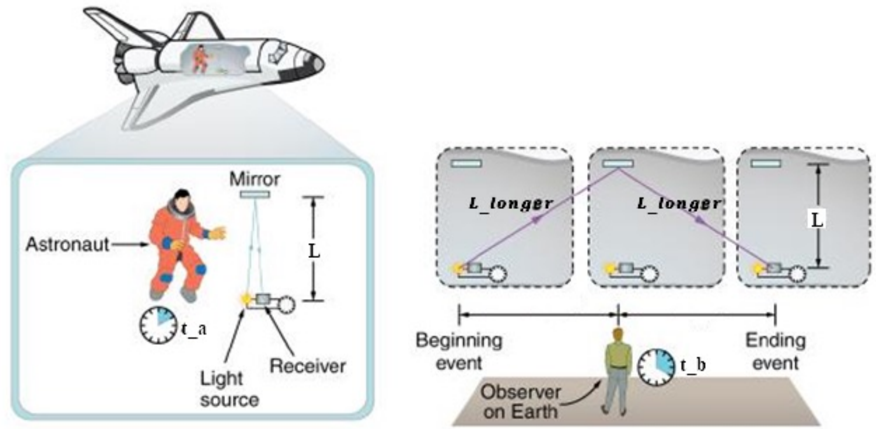
- Ball bounces to travel different distances in two cases (a) & (b)
- Case (a) Time of travel between bounces $\boldsymbol{t_a = \frac{2L}{c_B}}$
- Case (b) for the bouncing ball, total velocity will be greater: vector sum $c_{res} = c_B + v_{trolley}$
- Case (b) When moving, $\boldsymbol{t_b = \frac{2L_{longer}}{c_{res}}}$; distance measured by stationary observer is greater but so is the velocity observed by stationary observer.
- Hence, time remains same ($\boldsymbol{t_a = t_b}$) when distance increases

Special Relativity Case



- **Light** travels different distances in these two reference frames (a) and (b)
- (a) Time of travel between bounces $\boldsymbol{t_a = \frac{2L}{c}}$ (t_a is the Proper time t')
- (b) From moving reference frame, $\boldsymbol{t_b = \frac{2L_{longer}}{c}}$;
- For light, there is not vector sum so $c + v = c$ (2[nd] postulate)
- Hence, to cover more distance with same speed c you need ***more time*** $\Rightarrow \boldsymbol{t_b > t_a}$ $[t > t']$
- Time in (relatively) moving FOR (a), when measured by an observer in RF (b) passes slower, compared to time as measured by observer in FOR (a) (proper time)
- **Improper time interval is greater than the proper time interval**





Galilean relativity

- Time in an inertial FOR measured by an external fixed observer remains same as when measured by the moving observer, since measured velocity of object can change.
- $t = t_b \;\; [t' = t]$

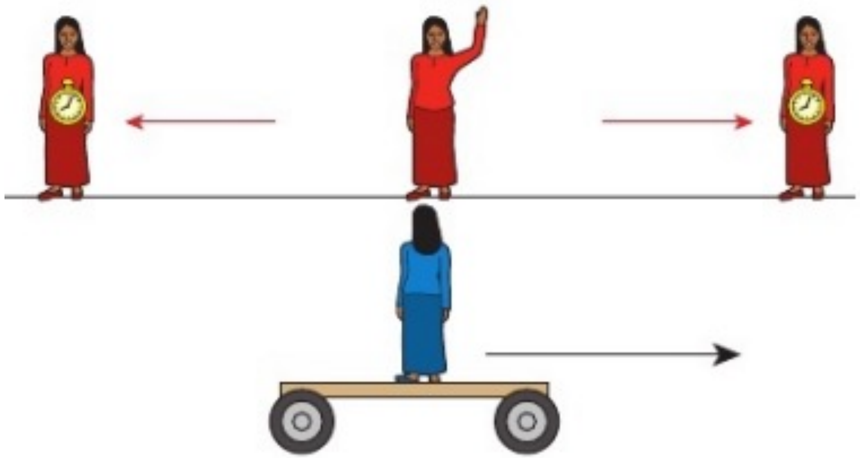
Special relativity

- Time in an inertial (moving) FOR (a), when measured by an observer in (stationary) FOR (b), passes slower compared to time measured in RF (a)
- $t_a < t_b$ (proper time $[t' < t]$)

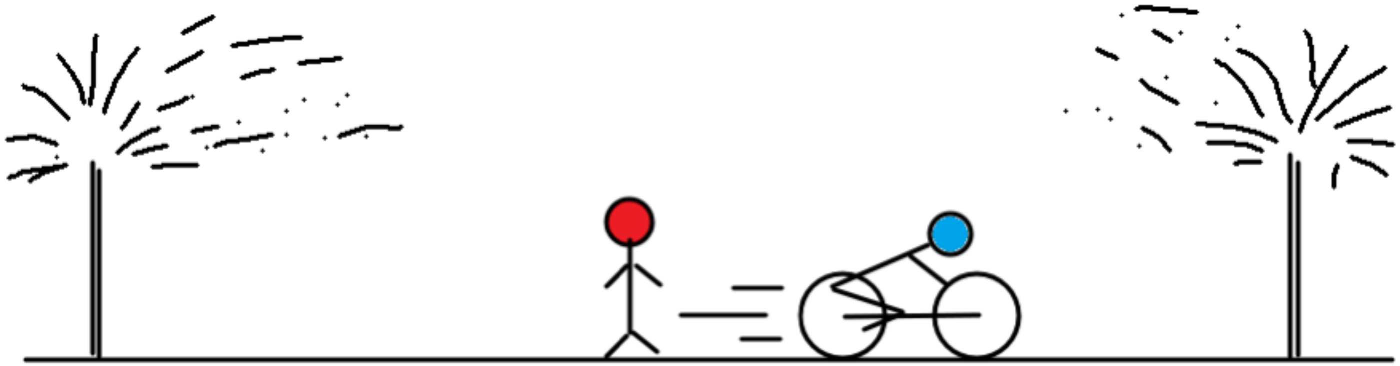
The problem of synchronization

- $Events\ that\ are\ synchronized\ in\ one\ frame\ of\ reference\ are\ not\ synchronized\ in\ the\ other.$

- Time is synchronized since light from your hand reaches both observers at the same time. Their clocks start at the same time.



- ***For the moving observer,*** left distance is more, since left is (relatively) moving in the direction of light itself
- And right distance is less since light is moving towards right
- Hence, their clocks start at different times

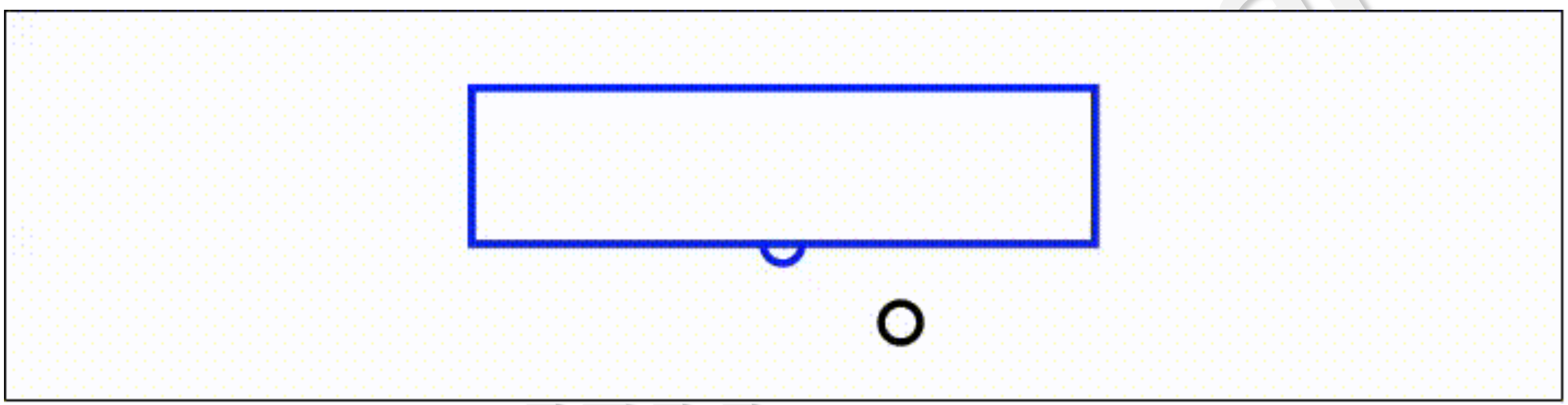


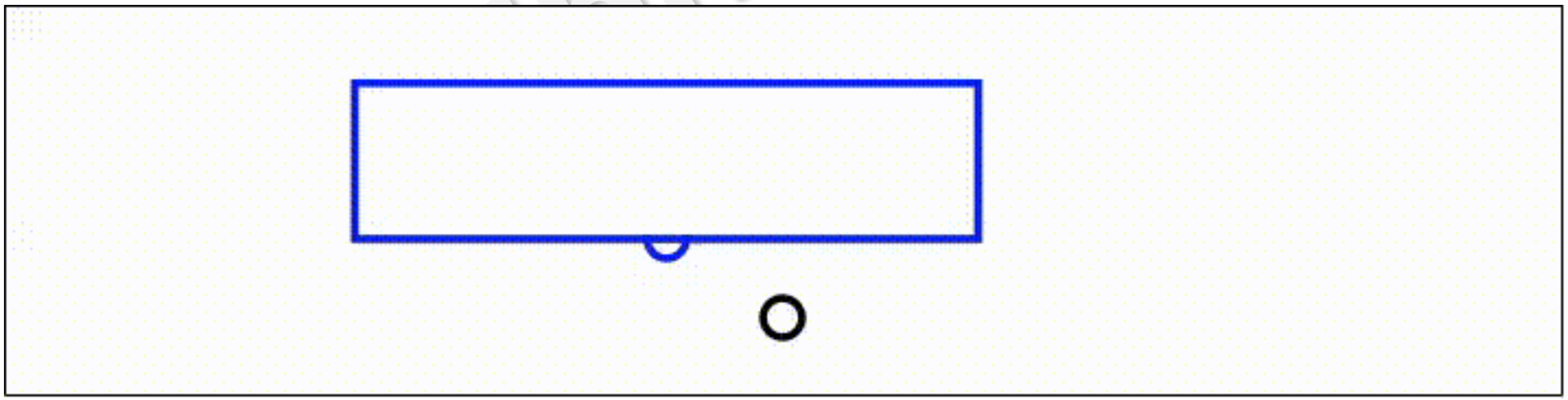
Simultaneity: An Analogy

- Think of two sprinklers separated by some distance & **switched on at the same time (simultaneously)**.
- If a person stands **exactly in the middle**, and another one travels **on a motorcycle** (at reasonable speed) between them,
- Which sprinkler's water will get the stationary person wet first?
- Which sprinkler's water will get the person on bike wet first?

- Problem with STR's simultaneity is that we fail to understand that **no information** reaches observer, "before he knows there is information".
- Unlike in sprinkler case, we know water's position/velocity etc. even before the water gets them wet.

Simultaneity: From different FOR





Transformations in Einstein's relativity

- Since synchronization is not possible in moving frames, the only thing we can do, is find a way to transform measurements from one frame to other as measured by them individually.
- Galilean transforms don't work for (speed of) light
- Transforms in Einstein's relativity are called the "Lorentzian transforms"
- Lorentzian transform is multiplicative factor $\boldsymbol{\gamma}$:

$$\boldsymbol{\gamma} = \frac{\mathbf{1}}{\sqrt{\mathbf{1} - \frac{\boldsymbol{v}^2}{\boldsymbol{c}^2}}}$$

- Dimensionless,

[28.2: Simultaneity and Time Dilation - Physics LibreTexts]

https://byjus.com/physics/derivation-of-lorentz-transformation/

- v is velocity of frame B with respect to (wrt) A, c: speed of light

Galilean Relativity

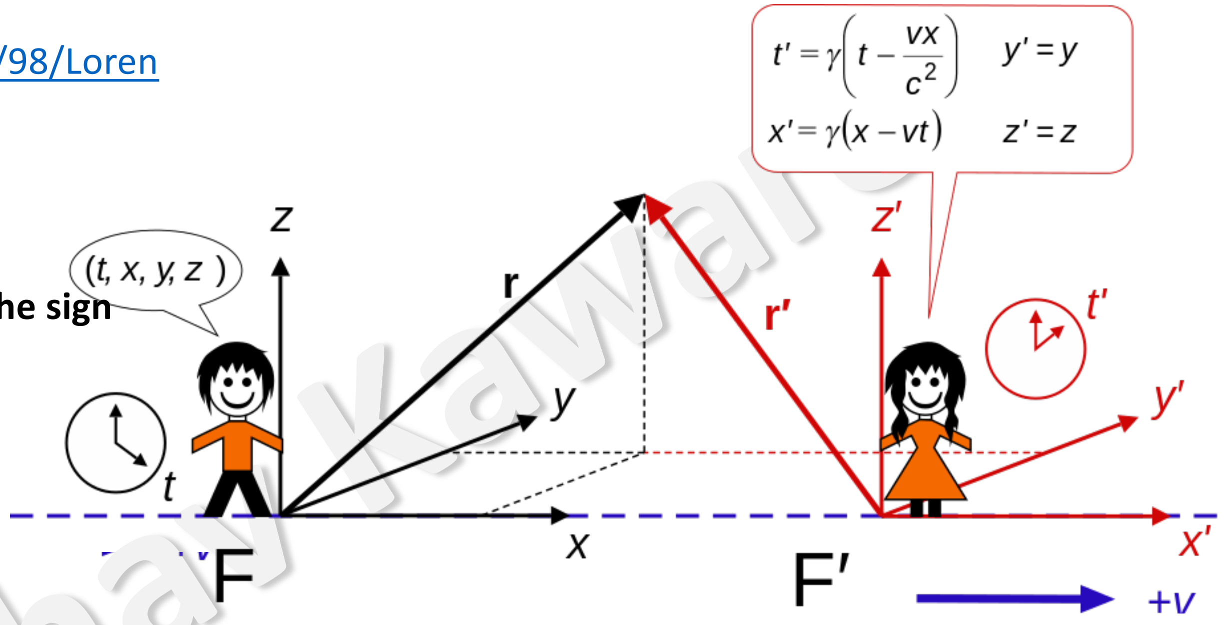
- $\because x = x' + vt$
- $\therefore$ Moving frame position $x' = x - vt$
- Length $\Delta x' = \Delta x$
- Time instant $t' = t$
- Time interval $\Delta t' = \Delta t$

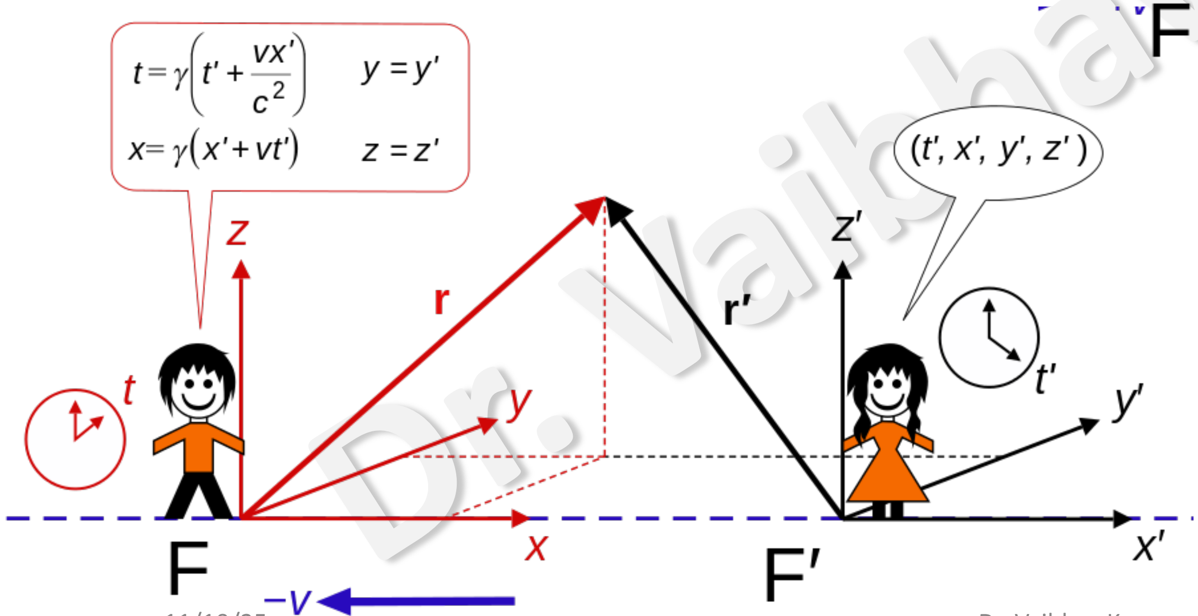
Einstein's Special Relativity

- $x' = \boldsymbol{\gamma}(x - vt)$
- $\Delta x' = \gamma(\Delta x - v\,\Delta t)$
- $t' = \gamma\left(t - \frac{vx}{c^2}\right)$
- $\Delta t' = \gamma\left(\Delta t - \frac{v\,\Delta x}{c^2}\right)$

https://upload.wikimedia.org/wikipedia/commons/9/98/Lorentz_boost_x_direction_standard_configuration.svg

To revert the transform equations,
Swap primed and unprimed quantities and reverse the sign





Data Booklet

x’:position of an event in an inertial frame of reference moving with relative speed v to the original frame of reference

x:position of the same event in the original frame of reference

v: relative speed between the two inertial frames of reference

t’: time of an event in an inertial frame of reference moving with relative speed v to the original frame of reference

t: time of the same event in the original frame of reference

u’: velocity of body in n inertial frame of reference moving with relative speed v to the original frame of reference

u: velocity of the same body in the original frame of reference

γ: the Lorrentz factor

c: speed of light in vacuum (constant)

$$x' = x - vt$$

$$t' = t$$

$$u' = u - v$$

Galilean

$$x' = \gamma(x - vt) \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

x':position of an event in an inertial frame of reference moving with relative speed v to the original frame of reference

x:position of the same event in the original frame of reference

v: relative speed between the two inertial frames of reference

t': time of an event in an inertial frame of reference moving with relative speed v to the original frame of reference

t: time of the same event in the original frame of reference

u': velocity of body in n inertial frame of reference moving with relative speed v to the original frame of reference

u: velocity of the same body in the original frame of reference

γ: the Lorrentz factor

c: speed of light in vacuum (constant)

Data Booklet

$$x' = x - vt$$

$$t' = t$$

$$u' = u - v$$

Formulae with a c^2 and/or γ are Relativistic, the others are Galilean

$$x' = \gamma(x - vt) \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

Two inertial frames, S and S′, coincident at time 0 s, move apart with relative velocity 0.9c as shown in Figure 18. An observer in S sees a balloon pop at $x = 5$ m at time 10^{-8} s. When and where did the balloon pop as measured by an observer in S′?

The relative speed of the two reference frames = 0.9c so:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.9^2c^2}{c^2}}} = 2.3$$

$$x' = \gamma(x - vt) \text{ where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Using the Lorentz transform for x:

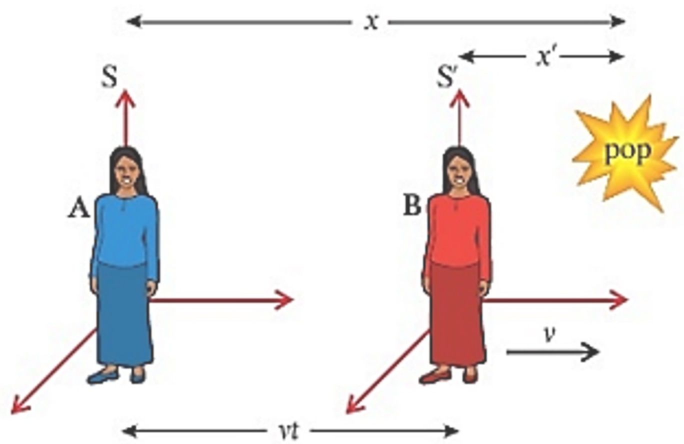
$$x' = \gamma(x - vt) = 2.3(5 - 0.9 \times 3 \times 10^8 \times 10^{-8}) = 5.29\,\text{m}$$

and for t:

$$t' = \gamma(t - \frac{vx}{c^2}) = 2.3(10^{-8} - \frac{0.9c \times 5}{c^2}) = -1.15 \times 10^{-8}\,\text{s}$$

This is before the clocks were started.

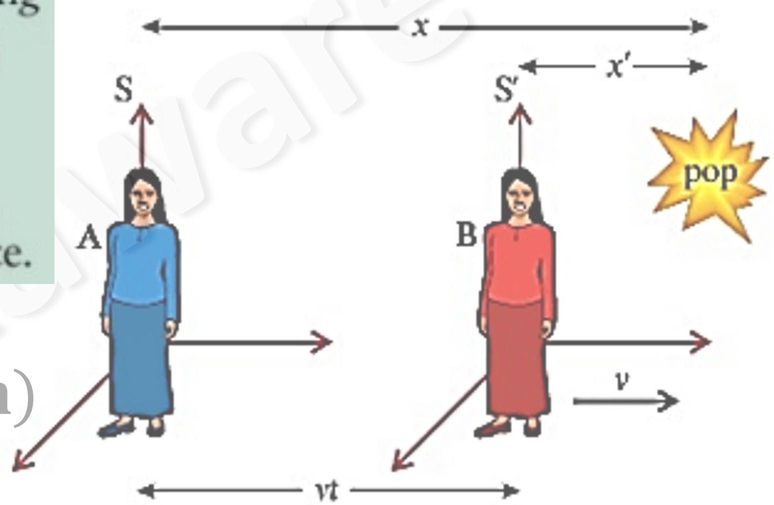
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$



An event takes place at a position of $x = 100$ m at a time 4×10^{-8} s as measured by an observer in frame of reference S. A second observer traveling at a speed of 2×10^8 m s^{-1} relative to the first along the line of the x-axis also measures the position and time for the event.

(a) Calculate the Lorentz factor between the two reference frames.

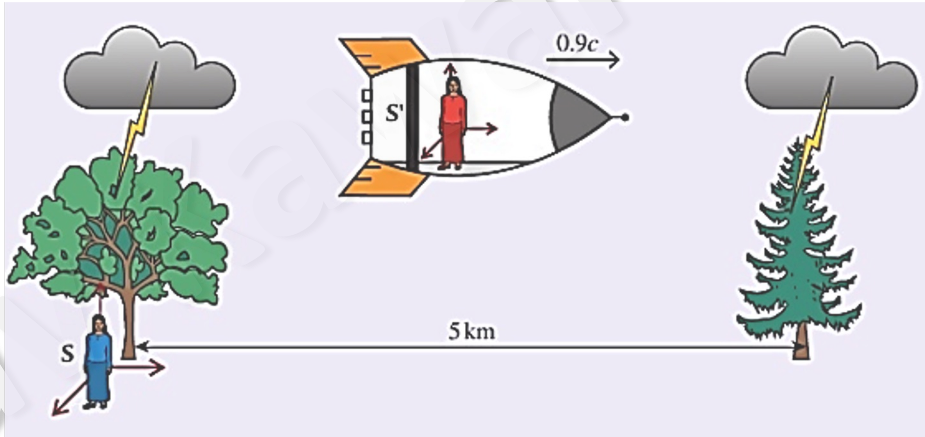
(b) Calculate the time and position measured in the second frame of reference.



- $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{(2\times10^8)^2}{(3\times10^8)^2}\right)}} = \frac{1}{\sqrt{1-\frac{4}{9}}} = 1.34$ …… (a)
- $x' = \gamma(x - vt) = 1.34\,[100 - (2\times10^8\times4\times10^{-8})] = 123.43\,m$ ……. (**b**)
- $t' = \gamma\left(t - \frac{vx}{c^2}\right) = 1.34\times\left[(4\times10^{-8}) - \left(\frac{2\times10^8\times100}{c^2}\right)\right]$
- $= 1.34\times\left[2\times\left(10^{-8} - \frac{10^8\times10^2}{9\times10^{16}}\right)\right] = 2.68\times\left(10^{-8} - \left(\frac{1}{9}\times10^{-6}\right)\right)$
- $= 2.68\times(0.01 - 0.11)\times10^{-6} = -5.69\times10^{-7}\,s$…. (**b**)

Simultaneity

- Two events which occur simultaneously in one frame, are not so in another moving reference frame.
- Lightening strikes simultaneously at $4\ \mu s$ on two trees 5 km apart
- For someone in rocket moving at $0.9\ c$ from tree 1 to 2 it won't be!
- Calculate time between the two events as observed from the spaceship.



- $t' = \gamma\left(t - \frac{vx}{c^2}\right)$; $\gamma = 2.3$; first lightning strike as observed from spaceship occurs at position $x = 0 \therefore t_1' = 2.3\times\left(4\ \mu s - \frac{0.9\ c\times 0}{c^2}\right) = 9.2\ \mu s$
- $t_2' = 2.3\times\left(4\ \mu s - \frac{0.9\ c\times(5\times 10^3)}{c^2}\right) = -25.4\ \mu s$
- To observer in the rocket, seconds tree was hit $\left(9.2 - (-25.4)\right) = 34.5\ \mu s$ earlier.
- Rocket travelling towards tree 2, hence light travels less time to the rocket ⇒ earlier

Repeat the Worked example above with the trees separated by 100 m and a speed of 0.8*c*.

- Lightening strikes simultaneously at $4\ \mu s$ on two trees $100\ m$ apart
- For someone in rocket moving at $0.8\ c$ from tree 1 to 2 it won't be!
- Calculate time between the two events as observed from the spaceship.
- $t' = \gamma\left(t - \frac{vx}{c^2}\right); \gamma = 1.67$
- first lightning strike as observed from spaceship occurs at position $x = 0$
- $\therefore t_1' = 1.67\times\left(4\ \mu s - \frac{0.8\ c\times 0}{c^2}\right) = 6.68\ \mu s$
- $t_2' = 1.67\times\left(4\ \mu s - \frac{0.8\ c\times(10^2)}{c^2}\right) = -15.5\ \mu s$
- To observer in the rocket, seconds tree was hit $(6.68 + 15.5) = 2.218\ \mu s$ earlier.
- Rocket travelling towards tree 2, hence light takes less time to travels to the rocket $\Rightarrow$ event occurs earlier

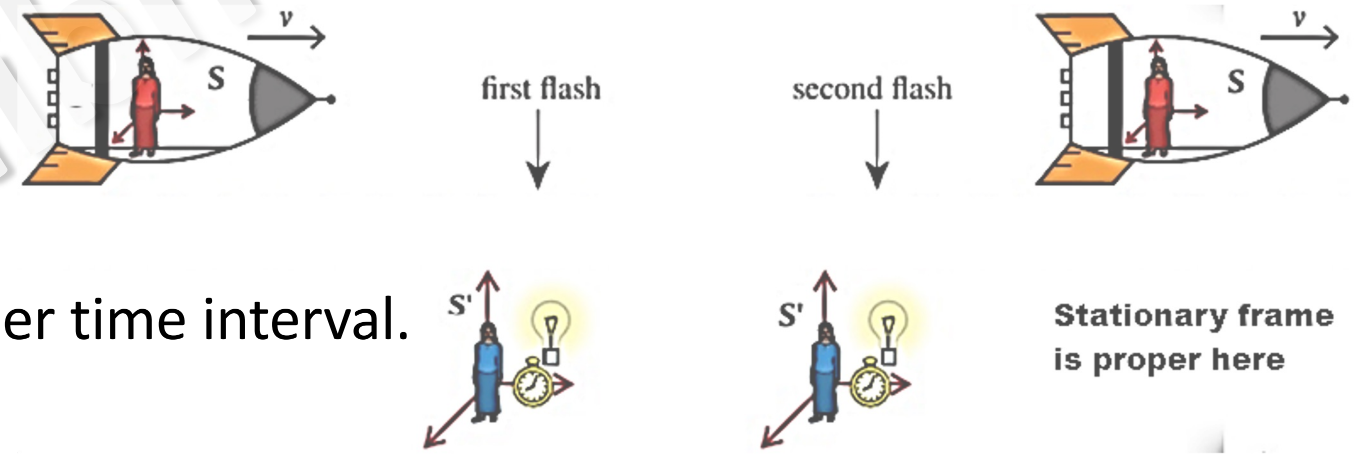
Time Dilation

- Light is switched on in FOR S' initially at time $t_1{}'$, and then again at time t_2' (***S'*** is the proper FOR)
- $\therefore \Delta t' = t_2{}' - t_1{}'$
- Other observer moving at v measures times t_1 & t_2. $\therefore \Delta t = t_2 - t_1$
- $t_1 = \gamma\left(t_1' + \frac{vx}{c^2}\right)$ & $t_2 = \gamma\left(t_2' + \frac{vx}{c^2}\right)$ $\therefore \Delta t = \gamma(t_2{}' - t_1{}')$
- Thus, intervals of time scale as $\boldsymbol{\Delta t = \gamma \Delta t'}$ (OR $\boldsymbol{T = \gamma T_0}$)
- Since $\gamma > 1$ always,

  time measured from external

  FOR will always

  be ***dilated (longer)*** than proper time interval.



A woman in a rocket claps her hands once every second as she flies past an observer on the Earth at a speed of 0.8*c*. What is the time between hand claps for the Earth observer?

- $\Delta t = 1\ s \therefore \Delta t = \gamma \Delta t'$
- $\gamma = 1.7\ ; \therefore \Delta t = 1.7\ s$

Two spaceships, A and B, pass in space at relative velocity 0.7*c*. An observer on A measures the time between swings of a pendulum he is holding to be 2 s. What will the time period be to an observer in B?

- $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.7^2)}} = \frac{1}{\sqrt{0.51}} = 1.4$
- $\Delta t = \gamma \times t'$
- $\quad = 1.4 \times 2 = 2.8\ s$

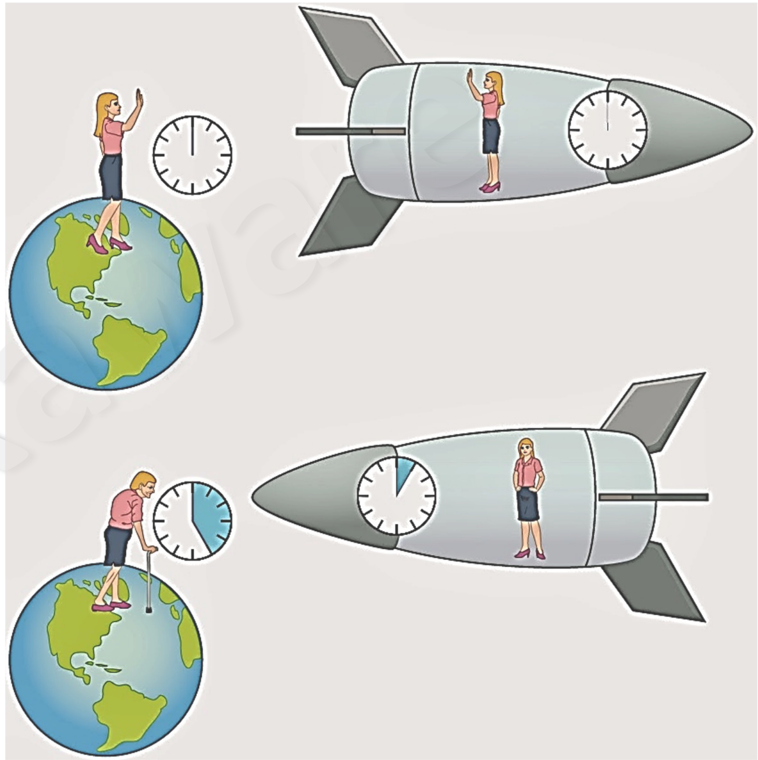
The half-life of the decay of some radioactive isotope is 30 s. The nucleus is accelerated to a speed of 0.99*c* relative to some observer. What will the half-life be to that observer?

- $\gamma = \frac{1}{\sqrt{1-0.99^2}} = 7.089$
- $T_{\frac{1}{2}} = 7.089 \times 30$
- $\quad = 212.7\ s$

A rocket travels between the Earth and some distant point at a constant speed of 0.8*c*. The time between these events is measured by an observer on the Earth and an observer on the rocket. The rocket observer measures the time to be 2 years.

(a) Which observer measures the proper time?

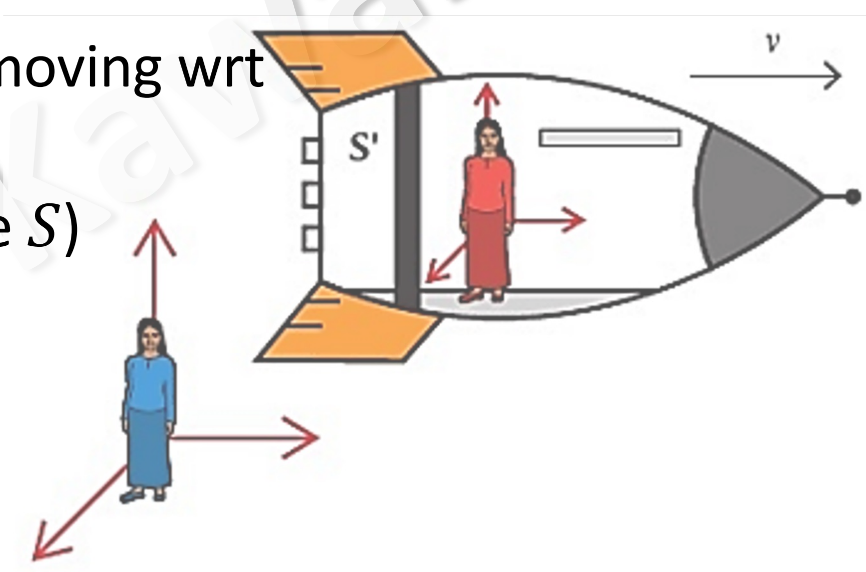
(b) What time will the Earth observer measure?



- (a) Proper time interval Δt is the time measured by an observer at rest relative to the event being observed.
- Hence, observer in the rocket measures the proper time $t' = 2\ Y$
- (b) $\gamma = \left(\frac{1}{\sqrt{1-0.8^2}}\right) = 1.667$
- $t = \gamma \times t' = 1.667 \times 2\ Y = 3.33\ Y$
- ***Simultaneous events occur at same time separated in space***
- ***Time dilation occurs at same position separated by time (time interval)***

Length contraction

- Length of the rod in moving FOR
- $\Delta x' = x_2' - x_1'$ Proper length, since rod is not moving wrt observer in S'
- For another observer (in stationary/fixed frame S)
- $x_1' = \gamma(x_1 - vt)$ & $x_2' = \gamma(x_2 - vt)$
- i.e., $\Delta x' = x_2' - x_1' = \gamma(x_2 - x_1) = \gamma\,\Delta x$
- $\boldsymbol{\Delta x = \frac{1}{\gamma}\Delta x'}$ OR $\boldsymbol{L = \frac{1}{\gamma} L_0}$
- Length measured from external frame appears contracted



Length contraction

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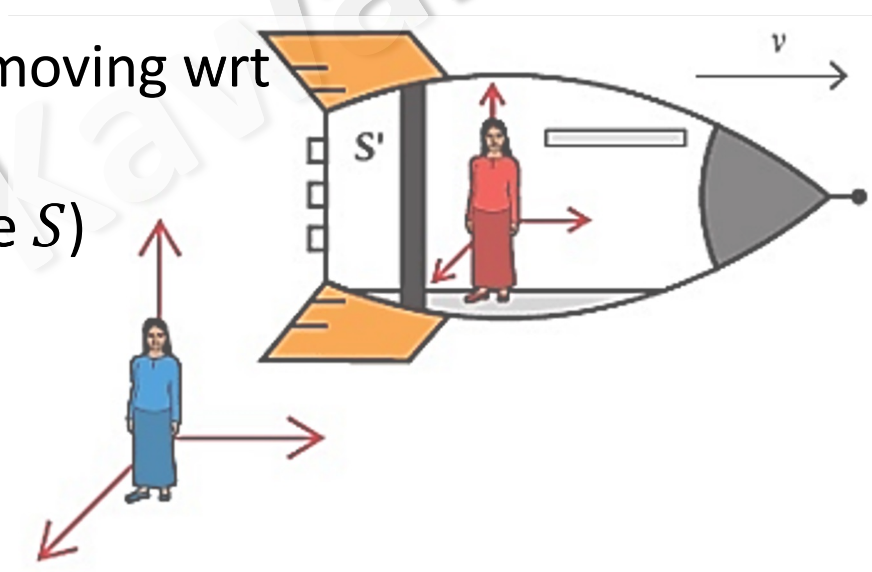
- $x_1' = \gamma(x_1 - vt)$ & $x_2' = \gamma(x_2 - vt)$
- i.e., $\Delta x' = x_2' - x_1' = \gamma(x_2 - x_1) = \gamma\, \Delta x$
- $\boldsymbol{\Delta x = \frac{1}{\gamma} \Delta x'}$ $\boldsymbol{OR\ L = \frac{1}{\gamma}\ L_0}$
- Length measured from external frame appears contracted



$$\boxed{L = \frac{L_0}{\gamma}}$$

L: observed length
L_o: proper length
γ: the Lorrentz factor

A 1 m ruler is lying next to an observer on the Earth. How long will the ruler be if measured by a second observer traveling at a constant velocity of $0.9c$ along the line of the ruler?

In this case, the proper length, $L_0 = 1$ m.

The relative speed of the two reference frames = $0.9c$ so:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.9^2 c^2}{c^2}}} = 2.3$$

According to the length contraction formula: $L = \frac{L_0}{\gamma} = \frac{1}{2.3} = 0.43$ m

An astronaut measures the length of an inanimate carbon rod on the space station to be 44 cm. What is the length of the rod measured by an observer flying past the space station at a relative speed of 0.83c (in a direction parallel to the rod)?

*Note: To measure a length, 2 position measurements must be made **at the same time** in the frame of reference

- Since rod is in the space station and the measurement is made in the space station itself, that is the proper length $L' = 44\ cm$,
- $L = \frac{L'}{\gamma} = \frac{L_0}{\gamma}$
- $\gamma = \left(\frac{1}{\sqrt{1-\frac{(0.83c)^2}{c^2}}}\right) = \frac{1}{\sqrt{1-0.83^2}} = 1.793$
- Hence, $L = \frac{44}{1.793} = 24.54\ cm$

$$\boxed{L = \frac{L_0}{\gamma}}$$

L: observed length
L_0: proper length
γ: the Lorrentz factor

Two spaceships, A and B, pass in space at relative velocity 0.7*c*. If an observer in B measures the length of a metal rod he is holding to be 2 m, what is the length of the rod as measured by an observer in A?

$$\boxed{L = \frac{L_0}{\gamma}}$$

L: observed length
L_o: proper length
γ: the Lorrentz factor

- $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.7^2)}} = \frac{1}{\sqrt{0.51}} = 1.4$
- Spaceship B is the proper system since rod and the observed are in spaceship B
- Proper length $L_0 = 2\ m$
- ∴ length as measured by observer in spaceship A is
- $L = \frac{L_0}{\gamma} = \frac{2}{1.4} = 1.43\ m$

A rocket travels to a distant point, fixed relative to the Earth, at a speed of 0.8*c*. The distance to the point measured by an observer on the Earth is 5 light hours (one light hour is the distance traveled by light in 1 hour).

(a) Calculate the time period of the flight measured by an observer on the Earth.

(b) Calculate the distance traveled as measured by an observer on the rocket.

(c) Calculate the time taken measured by the rocket observer.

Δt: time interval between two observed events (2 different clocks)

$$\Delta t = \gamma \Delta t_0$$

γ: the Lorrentz factor

Δt_0: proper time (time interval measured by same clock)

- **(c)** Improper time $\Delta t = L/v$
- $\Delta t = \frac{3\text{LH}}{0.8\ \text{c}} = \mathbf{3.75\ H}$
- *Cannot apply time dilation since events (of measuring the distance) do not occur at the same point n space.*

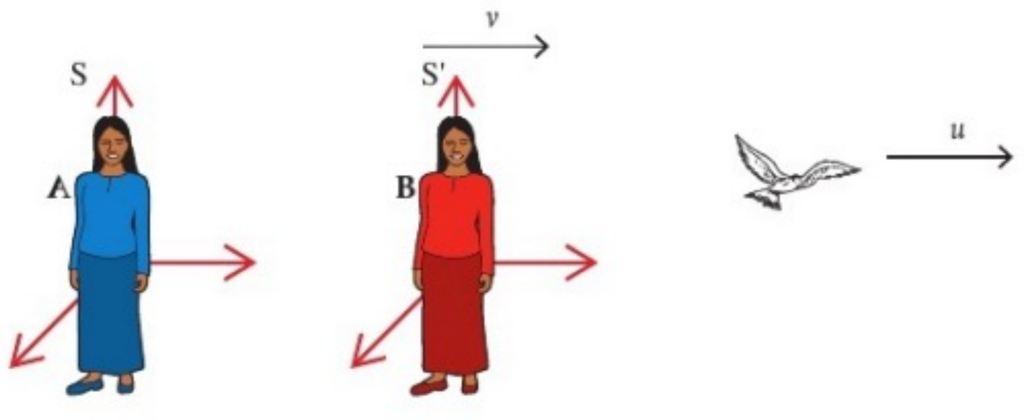
$$\boxed{L = \frac{L_0}{\gamma}}$$

L: observed length
L_0: proper length
γ: the Lorrentz factor

- $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.8^2)}} = 1.67$
- Earth is the proper system
- Hence **(a)** $\Delta t_{\text{earth}} = \frac{\text{distance}}{\text{speed}} = \frac{5\ LH}{0.8\ c}$
- $= \mathbf{6.25\ H}$
- **(b)** Here, $L_0 = L_{earth} = 5\ LH$
- ∴ Improper distance as measured from the rocket, $L = L_0/\gamma$
- $\therefore L = \frac{5}{1.67} = 2.99\ LH \approx 3\ LH$
- contracted

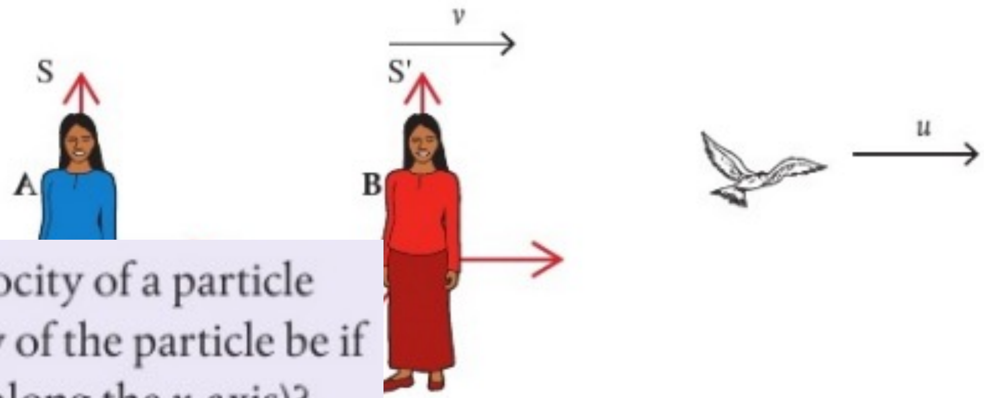
Velocity Lorentzian transform

- $\boldsymbol{u' = \frac{u-v}{1-\left(\frac{uv}{c^2}\right)}}$
- Here, u is the velocity measured from the stationary FOR, S
- … and u' the same velocity as measured from the moving FOR S'.



Velocity Lorentzian transform

- $u' = \dfrac{u-v}{1-\left(\frac{uv}{c^2}\right)}$



An observer in some frame of reference S measures the velocity of a particle moving along the x-axis to be $0.9c$. What would the velocity of the particle be if measured by an observer in S' moving at $0.5c$ relative to S (along the x-axis)?

Here we can simply substitute the values into the Lorentz velocity transformation:

$$u' = \frac{u-v}{1-\dfrac{uv}{c^2}}$$

where:

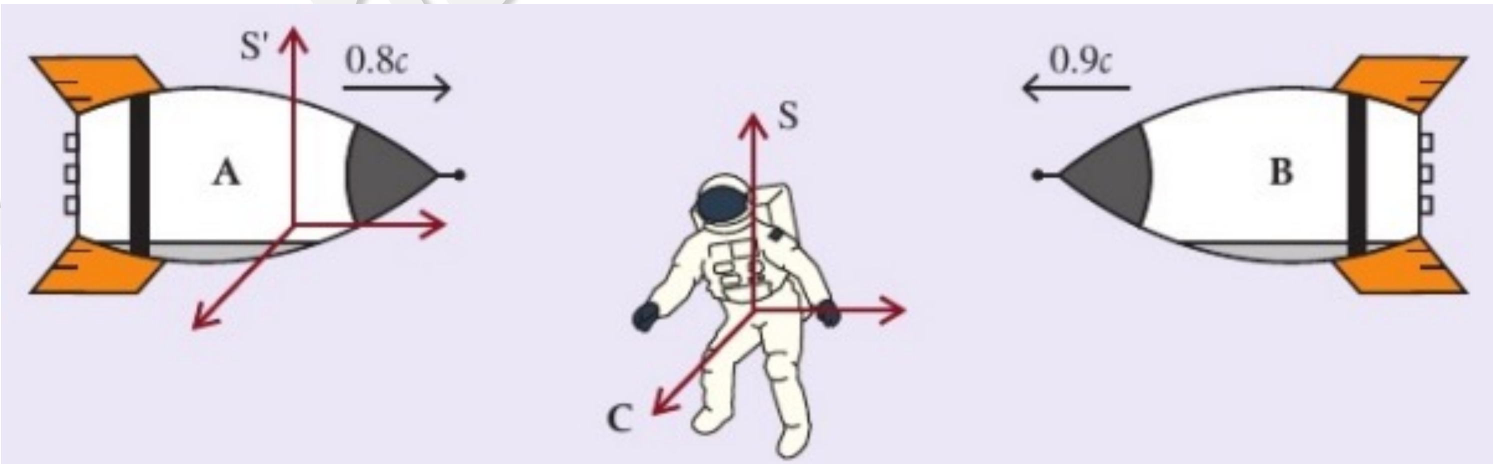
$u = 0.9c$

$v = 0.5c$

$$u' = \frac{0.9c - 0.5c}{1-\dfrac{0.9c \times 0.5c}{c^2}} = 0.7c$$

Two rockets approach an astronaut at speeds of 0.8c from the left and 0.9c from the right. At what speed will the rockets approach one another from the frame of reference of one of the rockets?

- Stationary observer: Astronaut (unprimed external observer)
- Assume one ship A ($0.8\ c$) as moving FOR (primed-proper FOR), and then find speed u' of the other ship B in FOR of first ship(moving FOR). Hence,
- $v = 0.8\ c$ & $u = -0.9\ c$ (B is moving to the left)
- $\boldsymbol{u' = \frac{u-v}{1-\left(\frac{uv}{c^2}\right)}}$
- $u' = \frac{(-0.9 - 0.8)c}{1-\left(\frac{-0.9\ c \times 0.8\ c}{c^2}\right)}$
- $u' = -\frac{1.7\ c}{0.22}$
- $u' = -0.988\ c$



Speed of light

- $\boldsymbol{u' = \frac{u-v}{1-\left(\frac{uv}{c^2}\right)}}$
- For $u = c$,
- $u' = \frac{\boldsymbol{c-v}}{\boldsymbol{1-\left(\frac{cv}{c^2}\right)}} = \frac{\boldsymbol{c-v}}{\frac{\boldsymbol{c^2-cv}}{\boldsymbol{c^2}}} = \frac{(c-v)}{\frac{\boldsymbol{c}(c-v)}{\boldsymbol{c^2}}} = \boldsymbol{c}$
- Thus, c is invariant under the Lorentzian transform (does not depend on v of the observer)

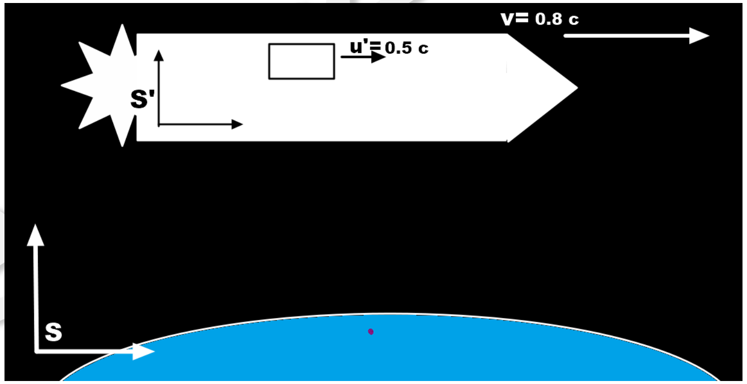
. Two subatomic particles are collided head on in a particle accelerator. Each particle has a velocity of 0.9*c* relative to the Earth. Calculate the velocity of one of the particles, as measured in the frame of reference of the other particle.

- Stationary FOR S is earth
- One of the particles is moving FOR, hence, $v = 0.9\ c$
- The other is moving in opposite direction to v with same speed, hence,
- $u = -0.9\ c$
- $\boldsymbol{u' = \frac{u - v}{1 - \left(\frac{uv}{c^2}\right)}}$
- $\therefore u' = \frac{-0.9c - 0.9c}{1 - \frac{(-)0.9c \times 0.9c}{c^2}} = -\frac{1.8\ c}{1 + 0.81} = -0.9945\ c$

. An observer on Earth sees a meteorite traveling at $0.5c$ on collision course with a spaceship traveling at $0.6c$. What is the velocity of the meteorite as measured by the spaceship?

- Earth: stationary FOR S (unprimed)
- Spaceship: Moving FOR S'; Hence, $v = 0.6\,c$ (v assumed to right)
- Meteorite 'approaches' spaceship hence, $u = -0.5\,c$
- To find velocity of meteorite u' as measured from spaceship
- $u' = \frac{u-v}{1-\left(\frac{uv}{c^2}\right)} = \frac{-0.5c-0.6c}{1-\frac{-0.5c\times0.6c}{c^2}} = -\frac{1.1c}{1-(-0.3)} = -\frac{1.1}{1.3}c = -0.8462\,c$

A spaceship is traveling at a speed of 0.80c relative to a ground station in frame S. A probe on the spaceship moves towards the front of the ship with a speed of 0.50c relative to the ship. What is the speed of the probe relative to the ground station?



- Moving frame S' is the spaceship.
- Velocity of the probe is with respect to (wrt) moving spaceship S$'$, hence "proper" velocity of probe is $u' = 0.5\ c$
- (In previous examples, both velocities were wrt ground S)
- To find u of the probe (as measured from stationary FOR - ground)
- Since $u' = \frac{u-v}{1-\left(\frac{uv}{c^2}\right)}$, reverting the velocity transform for u, we get

$$\boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}}$$

- $$u = \frac{u'+v}{1+\left(\frac{u'v}{c^2}\right)} = \frac{(0.5+0.8)c}{1+\frac{0.5c\times 0.8c}{c^2}} = \frac{1.3c}{1.4} = 0.93\ c$$

https://www.youtube.com/watch?v=JqwxQvq8IH0&list=PL2RRoMlng3gpCG4lWX1_BG-9D83KsKFBj&index=7 from CBS

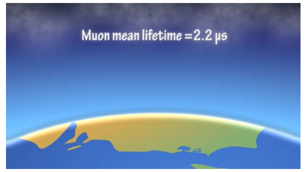
. A relativistic fly flies at 0.7*c* in the same direction as a car traveling at 0.8*c*. According to the driver of the car, how quickly will the fly approach the car?

- Earth: stationary FOR S
- Car: Moving FOR S'
- Hence, $v = 0.8\ c$
- Fly flies 'in the same direction as' spaceship hence, $u = 0.7\ c$
- To find: u'
- $u' = \frac{u-v}{1-\left(\frac{uv}{c^2}\right)} = \frac{0.7c-0.8c}{1-\frac{0.8c\times0.7c}{c^2}} = -\frac{0.1c}{1-0.56} = -0.2273\ c$
- Fly will be left behind the car at a relative speed of $-0.23\ c$.

What really travels near the speed of light?

- μ particles (muons) are produced in upper atmosphere ($\sim 10\ km$ above) when the (pions) π particles from the cosmic rays, decay. ($\pi \rightarrow \mu$)
- Speed of μ, $v = 0.98\ c$

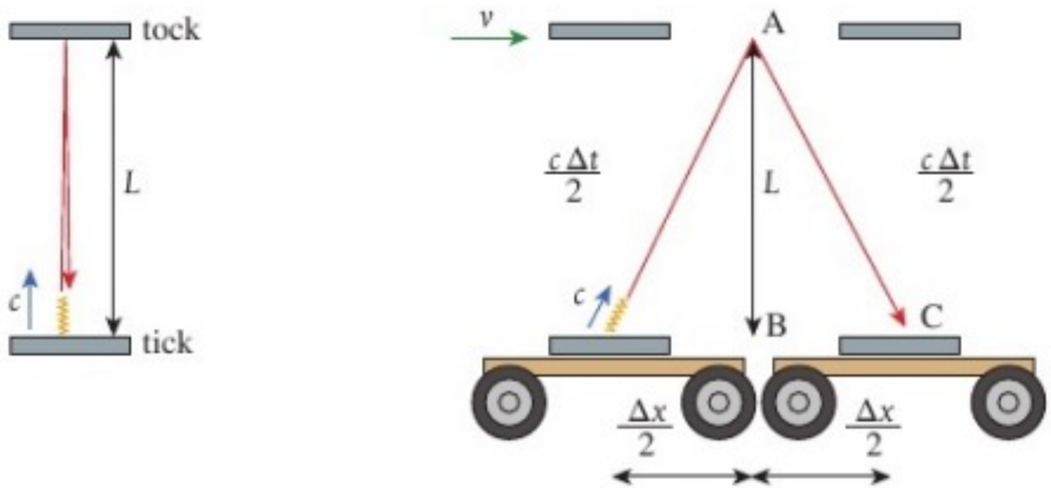
https://www.youtube.com/watch?v=nxQ0kz_qWmc&t=198s

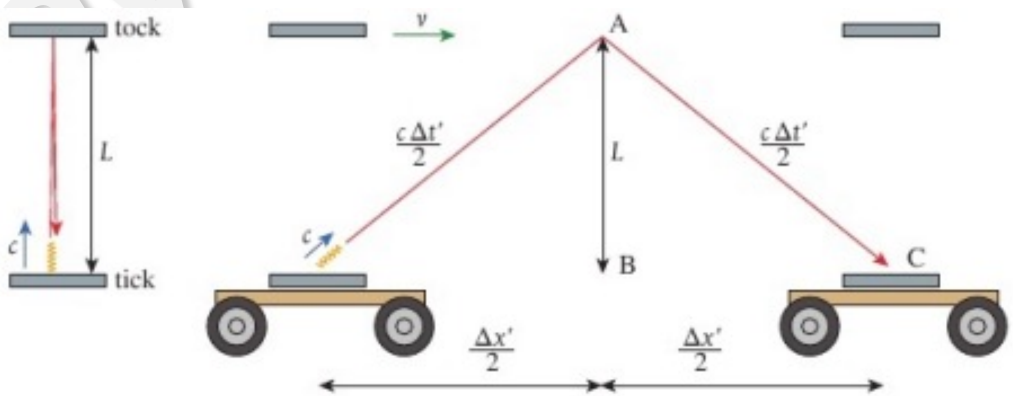


- Half-life $1.6\ \mu s \Rightarrow$ Half of initial number will decay in $1.6\ \mu s$.
- Stationary muon's mean lifetimes $2.2\ \mu s$
- Without STR, time required for muons to travel is $t = \frac{10^4\ m}{0.98\times(3\times10^8)} = 34\ \mu s$
- Number of muons reaching earth's surface from $h = 10\ km$ should be negligible, since time available for them to travel is much less than time required to travel $[2.2\ \mu s\ \&\ 1.6\ \mu s \ll 34\ \mu s]$
- With STR, $\gamma = 5$ for $v = 0.98\ c$
- From earth's FOR, time of travel to surface is dilated to $T = \gamma T_0 = 5\times1.6 = 8\ \mu s$
- From muon's FOR, length scales as $L = \frac{L_0}{5} = \frac{1}{5}\times10^4 = 2\ km$, and they will take $\frac{2\times10^3}{0.98\times(3\times10^8)} = 68\ \mu s$ much greater than time available $\mathrm{T}_{1/2} = 1.6\ \mu s$
- Hence, much more than half the initial number reach the ground

The space-time fabric

- The space-time interval is invariant in Einstein's relativity.
- Δt: Total time of light's travel
- By Pythagoras' theorem,
- $L^2 = \left(\frac{c\,\Delta t}{2}\right)^2 - \left(\frac{\Delta x}{2}\right)^2$
- For a different speed v,
- $L^2 = \left(\frac{c\,\Delta t'}{2}\right)^2 - \left(\frac{\Delta x'}{2}\right)^2$
- $\therefore \left(\frac{c\,\Delta t}{\mathbf{2}}\right)^2 - \left(\frac{\Delta x}{\mathbf{2}}\right)^2 = \left(\frac{c\,\Delta t'}{\mathbf{2}}\right)^2 - \left(\frac{\Delta x'}{\mathbf{2}}\right)^2$

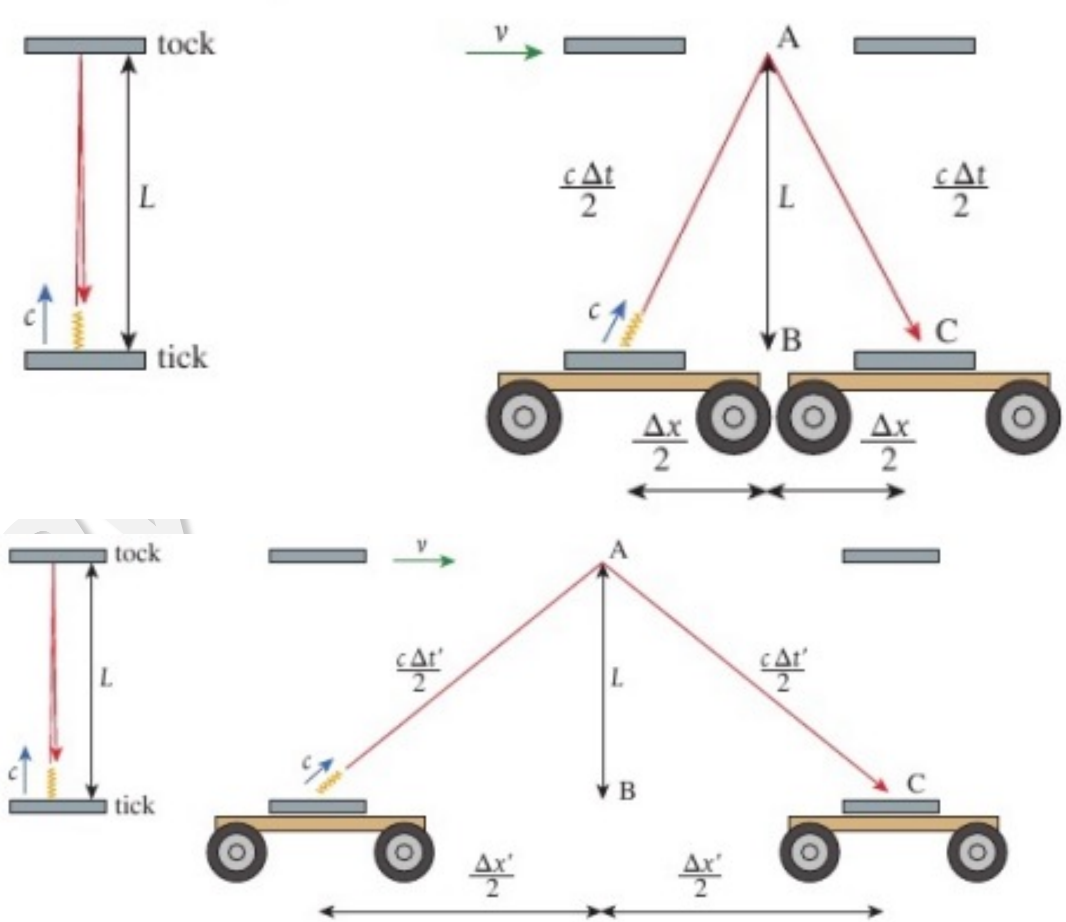




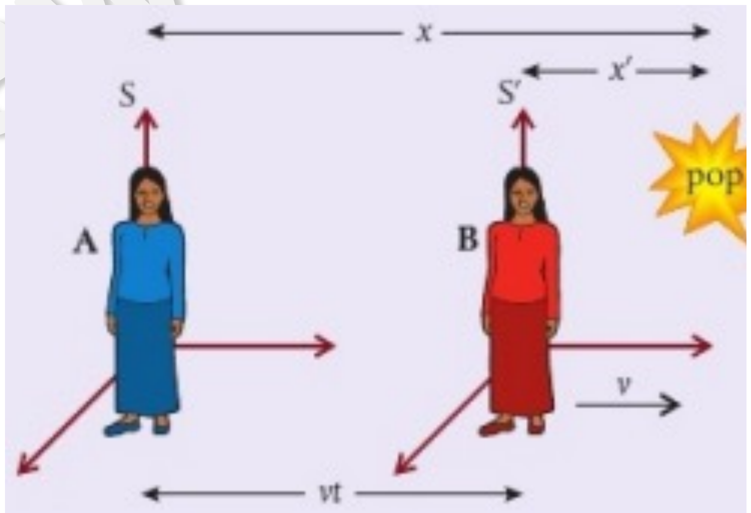
The space-time fabric

- The space-time interval is invariant in Einstein's relativity. $\therefore$
- $(c\Delta t)^2 - (\Delta x)^2 = (c\,\Delta t')^2 - (\Delta x')^2$

- Quantity $\boldsymbol{(ct)^2 - x^2}$ is the space-time ***interval (STI)***
- For stationary light clock, $\Delta x = 0$



- ***Reconsider***, two FOR S & S' coincide at $t = 0$ and then separate at $0.9\ c$ relative speed. Observer in S sees a balloon pop $x = 5\ m$ at time $t = 10^{-8} s$. Calculate the space-time interval ($\boldsymbol{STI}$) for each of the observer towards the event
- For $v = 0.9\ c, \gamma = 2.3$
- $x' = \gamma(x - vt) = 2.3(5 - (0.9 \times 3 \times 10^8 \times 10^{-8})$
- $x' = 5.28\ m$ & proper time t' (in frame $\mathbf{B}$)
- $t' = \gamma\left(t - \frac{vx}{c^2}\right) = 2.3\left(10^{-8} - \frac{0.9\ \boldsymbol{c} \times 5}{c^{\mathbf{2}}}\right)$
- $t' = -1.15 \times 10^{-8}\ s$
- $\therefore STI$ in S $S_{STI} = (ct)^2 - x^2$
  $= \left((3 \times 10^8 \times (10^{-8}))^2 - 5^2\right) = -16\ m^2$ &
- $S'_{STI} = (ct')^2 - (x')^2$
  $= \left((3 \times 10^8 \times (-1.15 \times 10^{-8}))^2 - 5.28^2\right) = -16\ m^2$



An event takes place at a position of $x = 100$ m at a time 4×10^{-8} s as measured by an observer in frame of reference S. A second observer traveling at a speed of $2 \times 10^{8}\ \mathrm{m\,s^{-1}}$ relative to the first along the line of the x-axis also measures the position and time for the event.

(a) Calculate the Lorentz factor between the two reference frames.

(b) Calculate the time and position measured in the second frame of reference.

- Calculate STIs for the above problem
- $S_{STI} = \left(\left((3\times10^{8})\times(4\times10^{-8})\right)^{2} - 100^{2}\right)$
- $S_{STI} = -9856\, m^{2}$

- $S_{STI} = \left(\left((3\times10^{8})\times(-5.69\times10^{-7})\right)^{2} - 123.43^{2}\right)$
- $S_{STI} = -9856\, m^{2}$

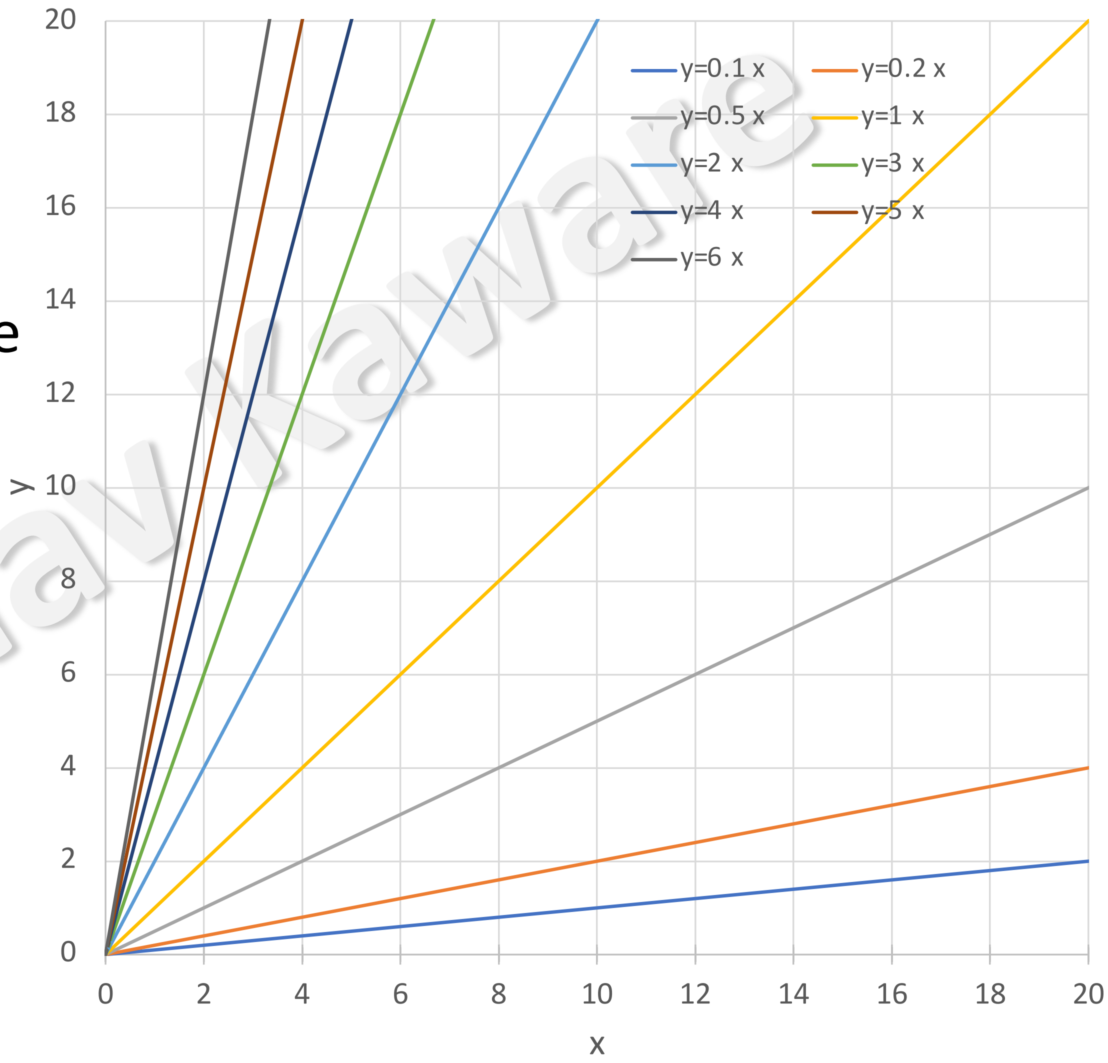
- $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{(2\times10^{8})^{2}}{(3\times10^{8})^{2}}\right)}} = \frac{1}{\sqrt{1-\frac{4}{9}}}$

  $\therefore \boldsymbol{\gamma = 1.342}$ …… **(a)**
- $x' = \gamma(x - vt) = 1.34\,[100 - (2\times10^{8}\times4\times10^{-8})]$
- $\therefore \boldsymbol{x' = 123.43\ m}$ ……. **(b)**
- $t' = \gamma\left(t - \frac{vx}{c^2}\right) = 1.34\times\left[(2\times10^{-8}) - \left(\frac{2\times10^{8}\times100}{c^2}\right)\right]$
- $= 1.34\times\left[2\times\left(10^{-8} - \frac{2\times10^{8}\times10^{2}}{9\times10^{16}}\right)\right]$
- $= 2.68\times\left(10^{-8} - \left(\frac{2}{9}\times10^{-6}\right)\right)$

  $\boldsymbol{t' = -2.44\times10^{-7}\ s}$…. **(b)**

Space-Time Diagrams (STD)

- **NOS:** Theory is easier to understand when expressed in diagrams
- **Space-time diagrams** are an alternative way to solve STR problems.
- **Space-time connection** is difficult to comprehend
- **Space-time diagrams** make is easier to visualize



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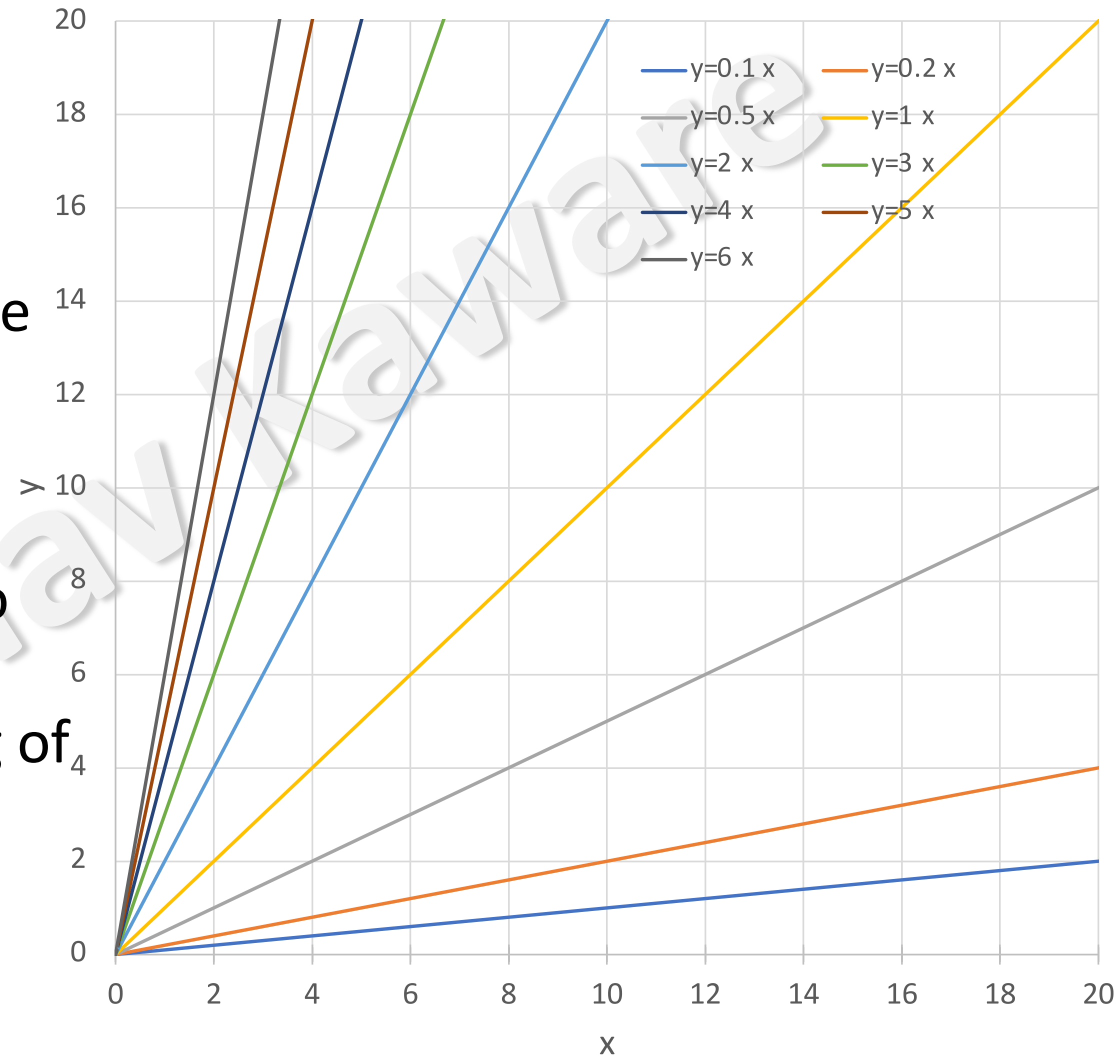
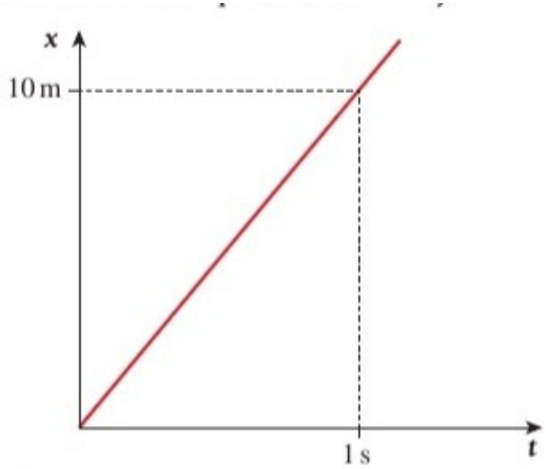
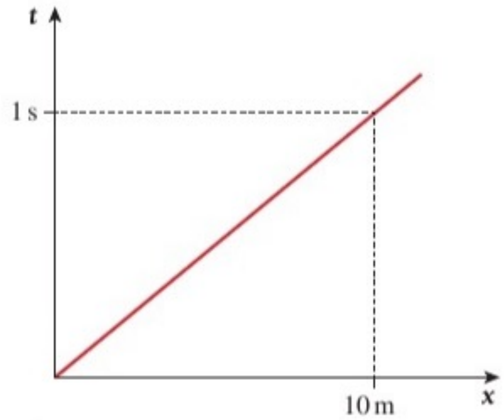
- These diagrams require understanding of graphs
- 1. Higher the gradient steeper the line



Space-Time Diagrams (STD)





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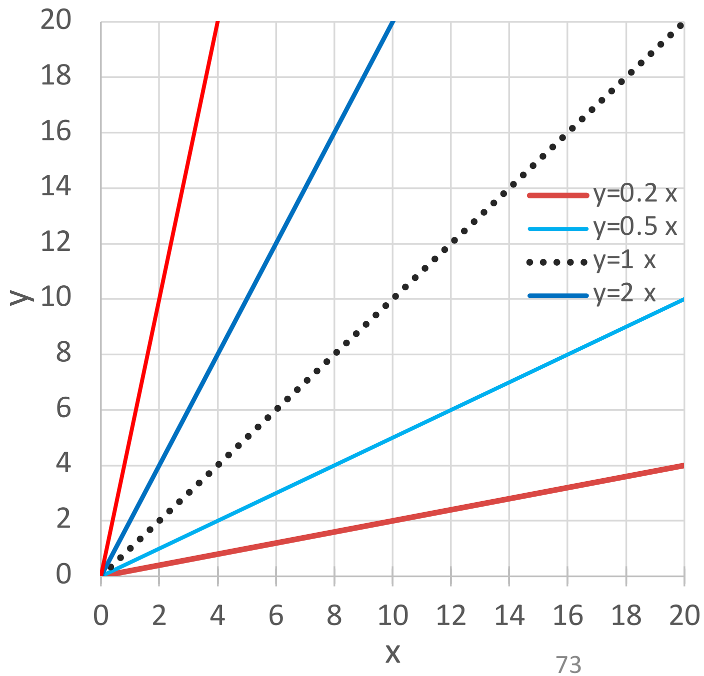
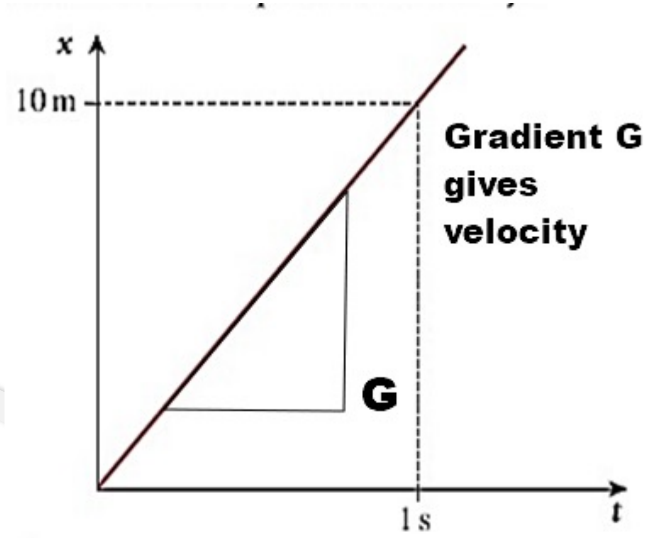
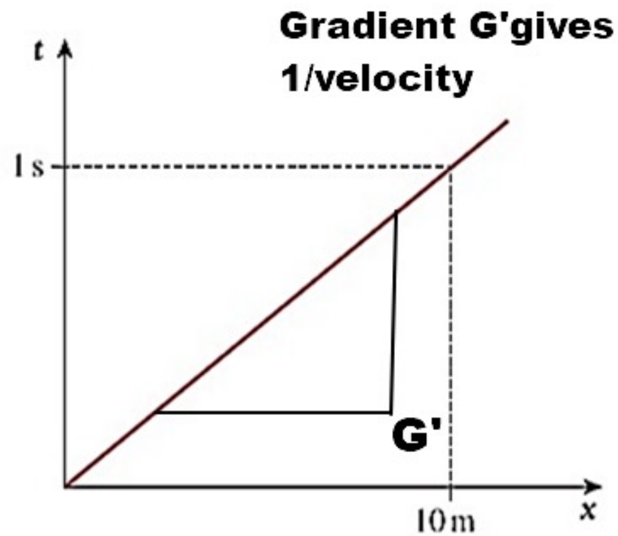
- These diagrams require understanding of graphs
- 1. Higher the gradient steeper the line
- 2. Exchange of axis reflects graph around 45° line
- 3. Gradient get reciprocated



Space-Time Diagrams (STD)

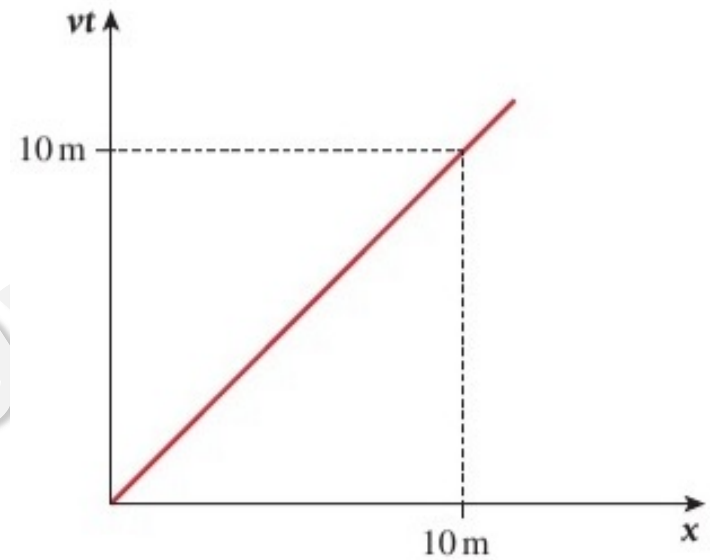
- Gradient of graph reciprocates when axes are exchanged
- $G = \frac{\Delta x}{\Delta t}$ velocity
- $G' = \frac{\Delta t}{\Delta x}$ 1/velocity & velocity= $\frac{1}{G'}$
- Gradient G has dimensions of velocity (ms^{-1})
- Gradient G' has dimensions $(1/ms^{-1})$
- Larger G' ⇒ lower velocity &
  Smaller G' ⇒ Higher velocity

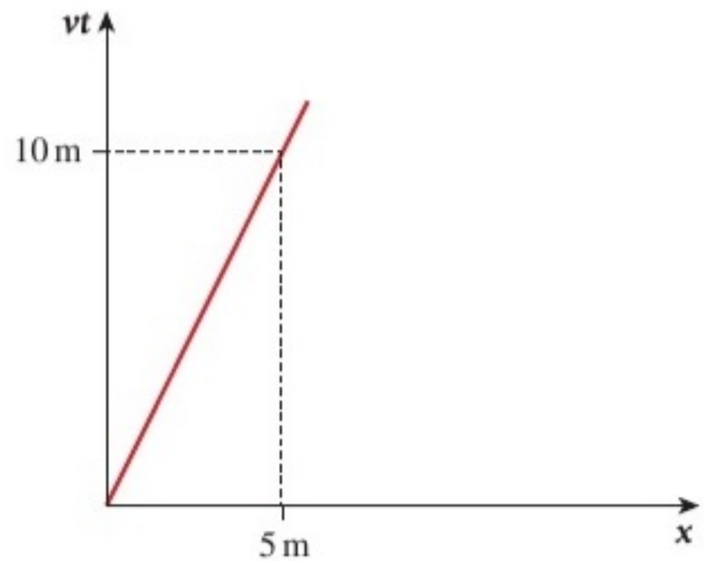




Space-Time Diagrams (STD)

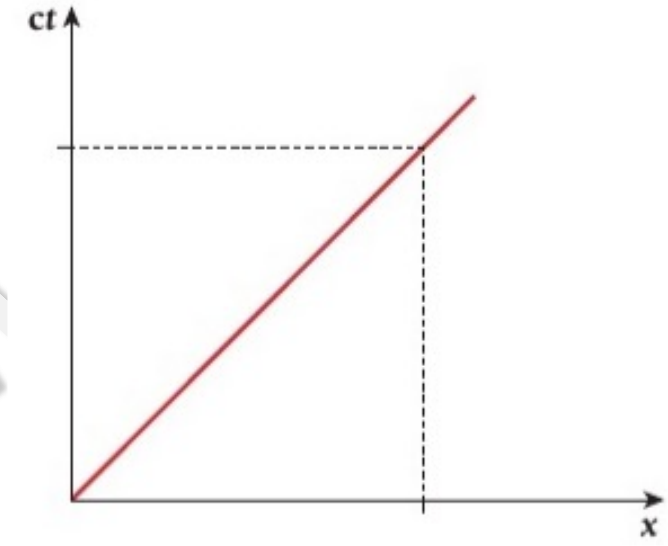
- Gradient G' becomes dimensionless if we plot vt instead of just t on the y axis
- gradients $= 1$ & 2 respectively
- Both graphs have gradients with no dimensions
- y-axis *is time* in terms of $speed$ $(\times time)$
- 2nd graph corresponds to **lower** velocity of object (higher gradient)
  (since time is on y-axis and displacement on x)

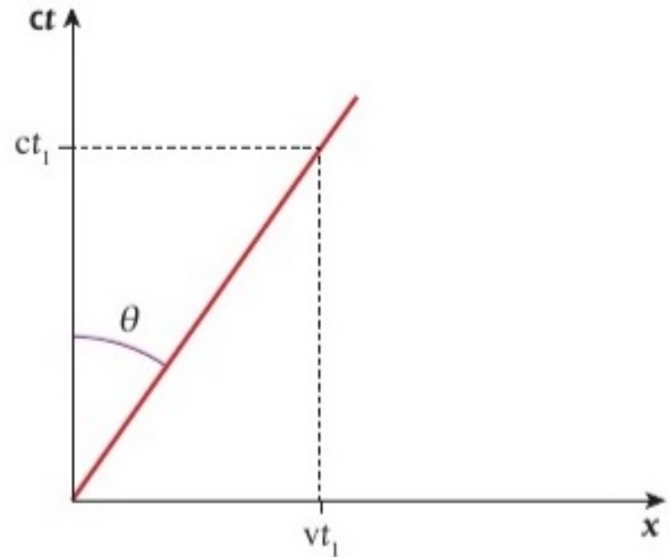




Space-Time Diagrams (STD)

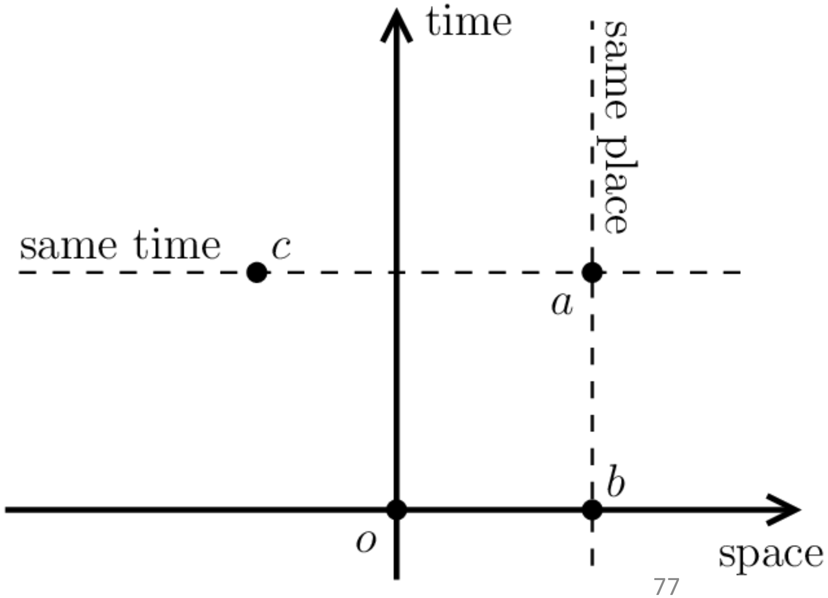
- In Relativity's Space-Time Diagram, y axis is ct instead of vt
- STD compares displacement of particle x $(= vt)$ with displacement of photon (ct)
- Photon has gradient $G = 1$ on this graph
- The space-time line is called $worldline$
- All speeds of objects have worldlines 'above' the gradient $G = 1$ (photon's) worldline ($v < c$ always)
- In time t_1 object travels distance vt_1 (x-axis) and in same time, photon would have travelled distance ct_1
- Gradient of worldline is $tan\,\theta = \frac{vt_1}{ct_1} = \frac{v}{c}$





Space Time Diagrams (STD)

- Every point in STD is an event
- Vertical line ⇒ position fixed in space (all 3 dimensions)
- Horizontal line (fixed in time: imaginative, unreal)



Space Time Diagrams (STD)

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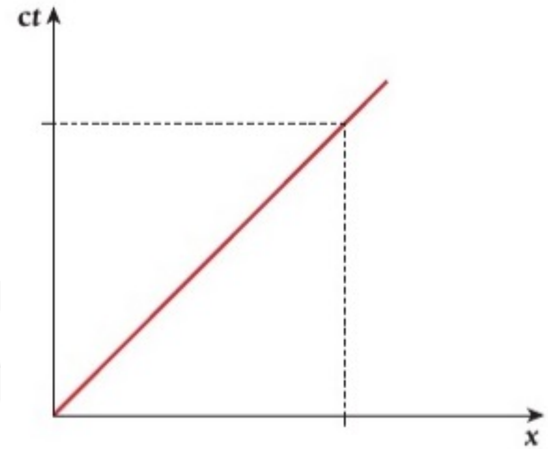
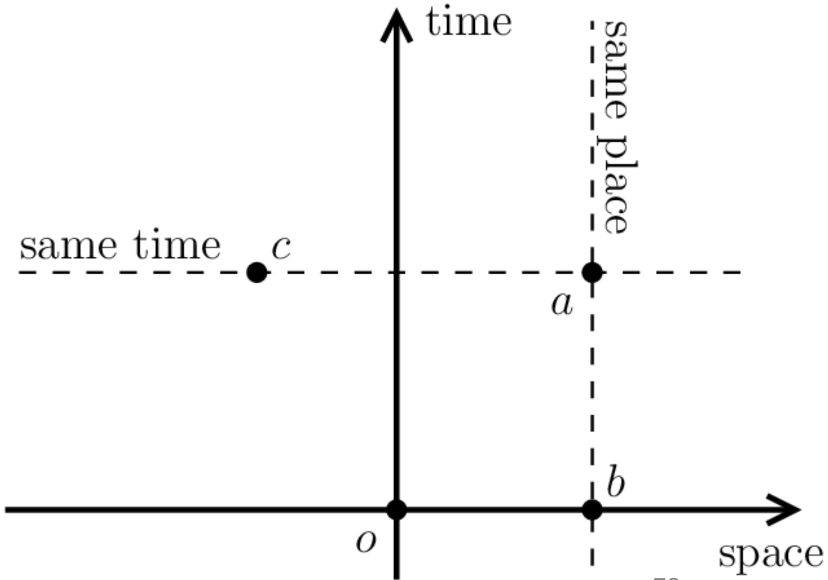
- **Single** straight line ⇒ motion (worldline) of one object





Space Time Diagrams (STD)

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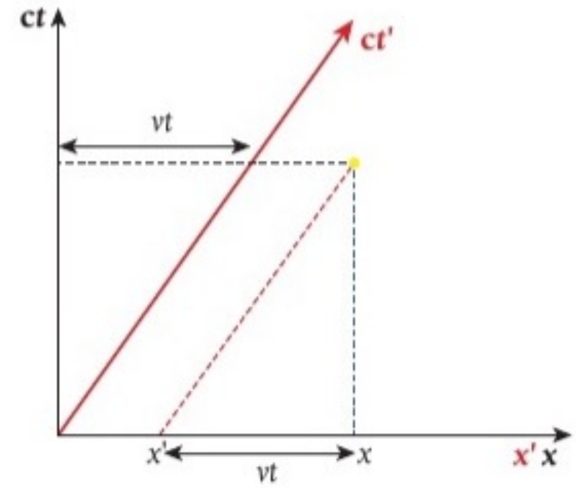
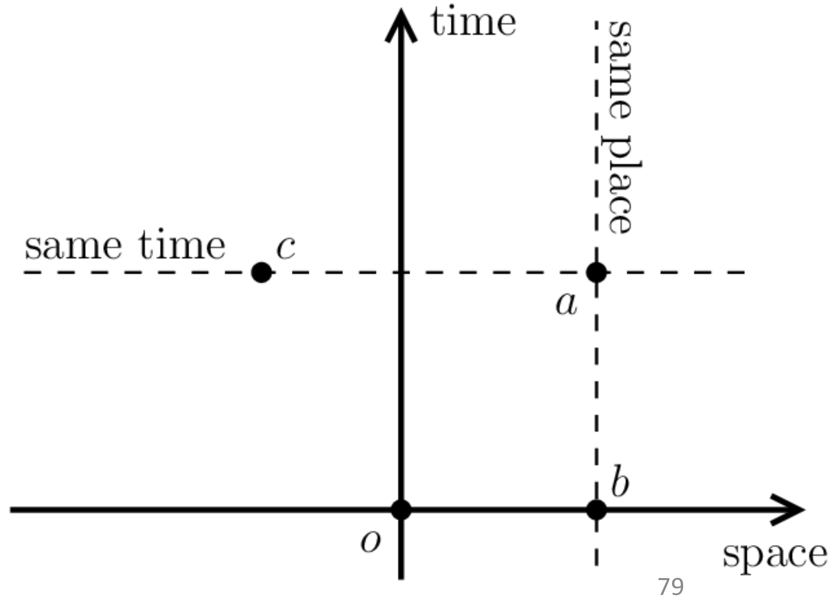
- Vertical line ⇒ position fixed in space (all 3 dimensions)
- Horizontal line (fixed in time: imaginative, unreal)
- **Single** straight line ⇒ motion (worldline) of one object
- **Two** straight lines ⇒ motion of two (worldline) objects
- **Single** set of axes ⇒ motion observed within the same FOR
- **Two** sets of axes ⇒ motion observed from different FOR



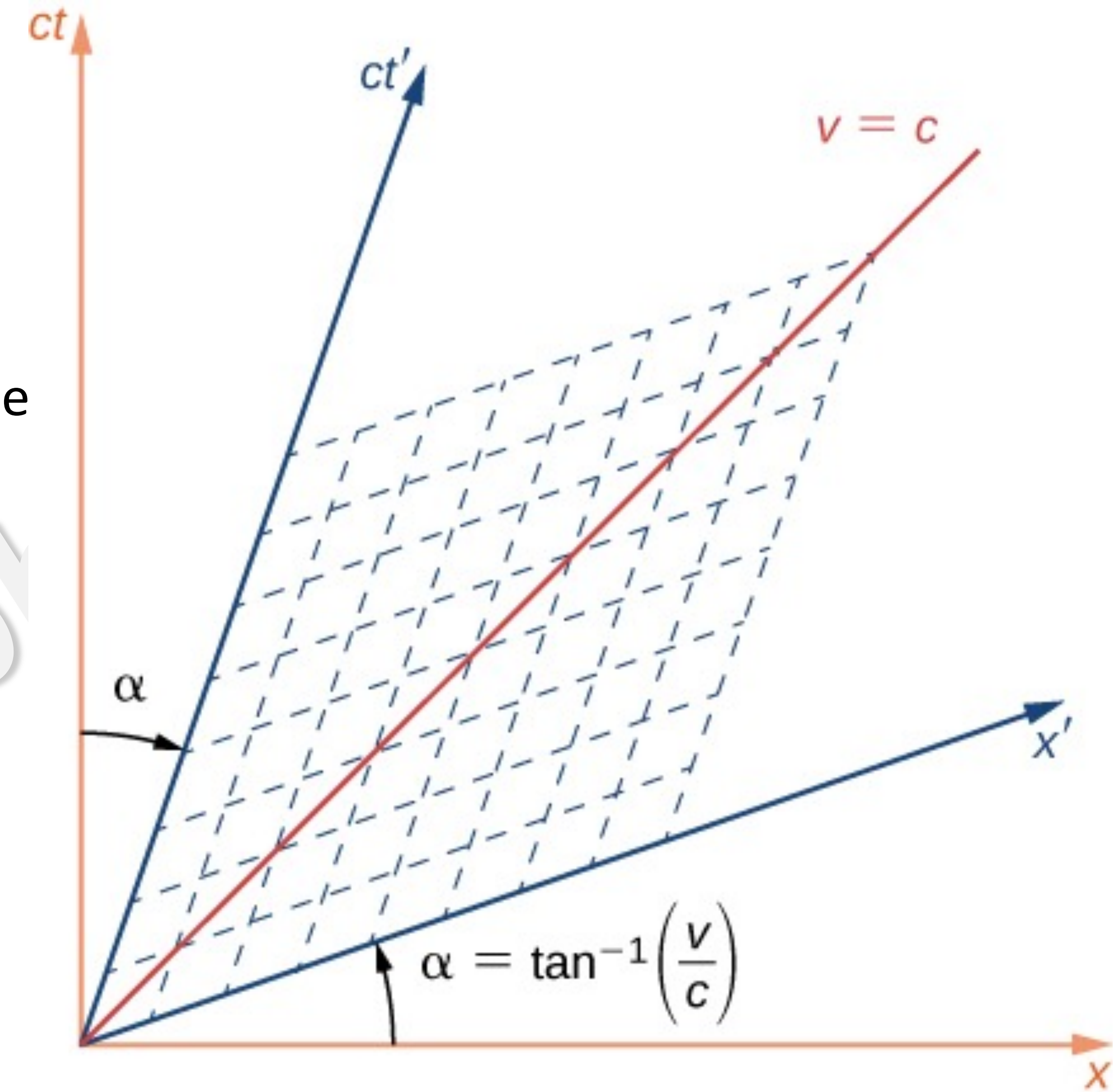


Summary: Space Time Diagram

- Visual representation of events in space at different times.
- It can show the geometry behind phenomena like time dilation and length contraction without using mathematical equations.
- Time ($\times c$) is represented on the vertical axis and space (3D) on the horizontal axis.
- The bottom of the diagram (3^{rd} & 4^{th} quadrants) represents the past, or early times & the top (1^{st} and 2^{nd} quadrants) represents the future, or later times.
- Each point on the graph, is called an event and an event represents a unique point in space and time.
- An object's world line represents the history of an object's location over time as a line or curve.
- The slope of the world line indicates how fast the object is moving
- **Faster** objects having **Less sloped** lines.

SPACE-TIME DIAGRAM (STD)

- STD has (at least) two sets of axes
- One belongs to fixed frame S with axes ct and x, and
- The other belongs to moving frame S' with axes ct' and x'
- We drop perpendiculars to get coordinates on the S axes, while
- We draw parallels to other axis to get required axis coordinates.



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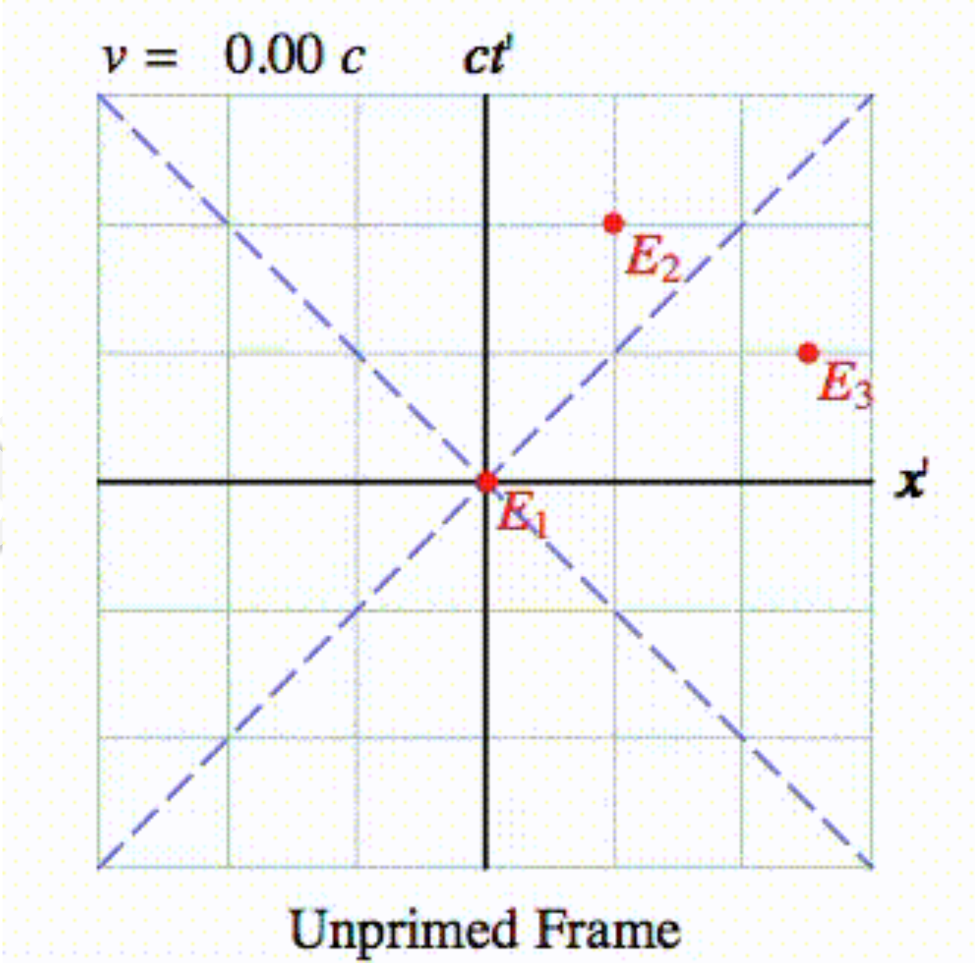

https://physics.nyu.edu/~ts2/Animation/special_relativity.html#

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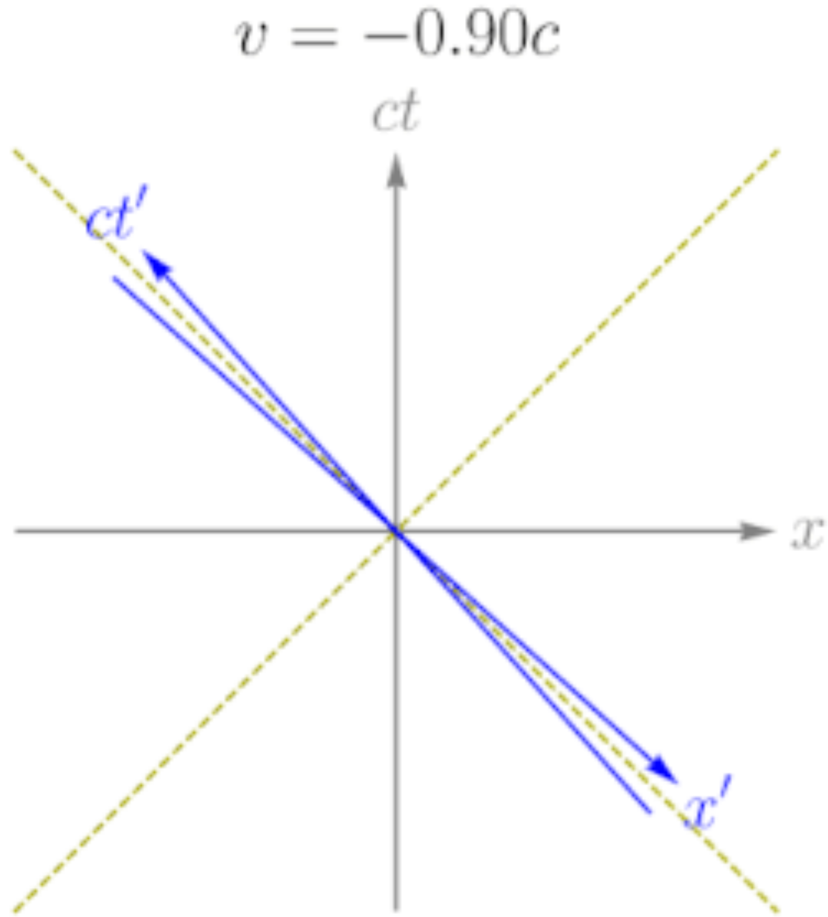
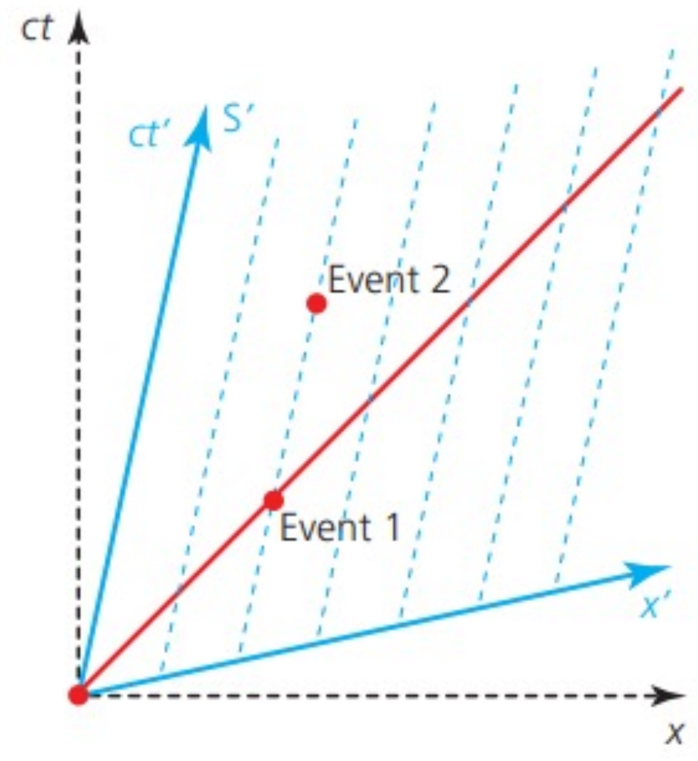


https://physics.nyu.edu/~ts2/Animation/special_relativity.html#

- Events that occur at same place as seen in frame S'
- Same event is not at the same place as seen from the other FOR S
- A person sitting in one place is not moving for people in train, but
- … is getting displaced as viewed by someone on the platform



■ **Figure A5.21** Space–time diagram with dashed blue lines representing separate world lines for different points that are each stationary in reference frame S'

- Events that occur at same place as seen in frame S'
- That event is not at the same place as seen from the other FOR S
- Events 3 and 4 occur at same time is FOR S' but not as seen from the other FOR S
- E.g. Loss of simultaneity is STR

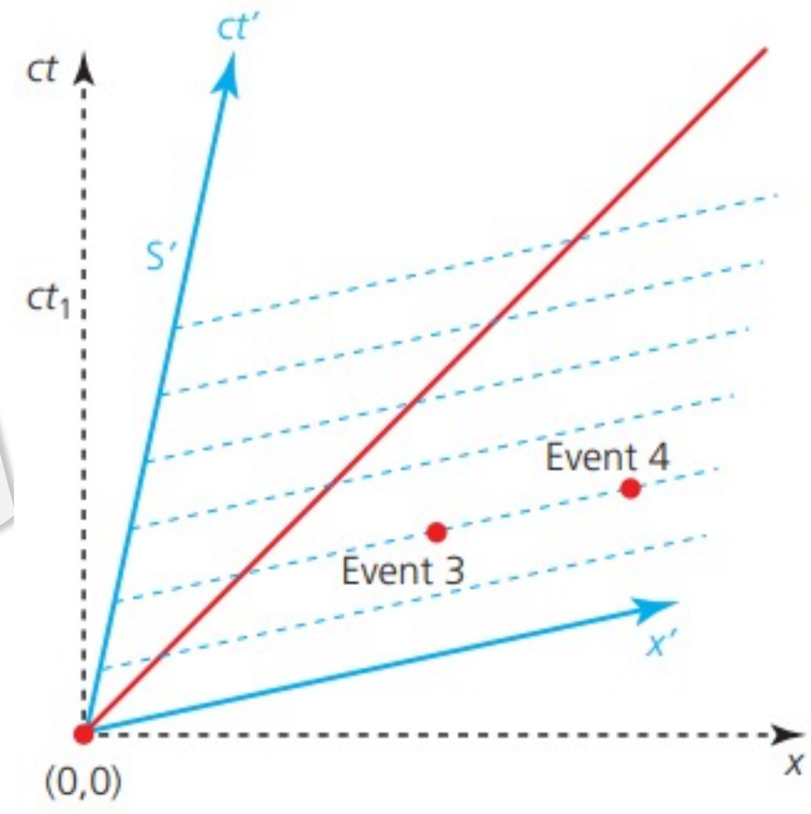
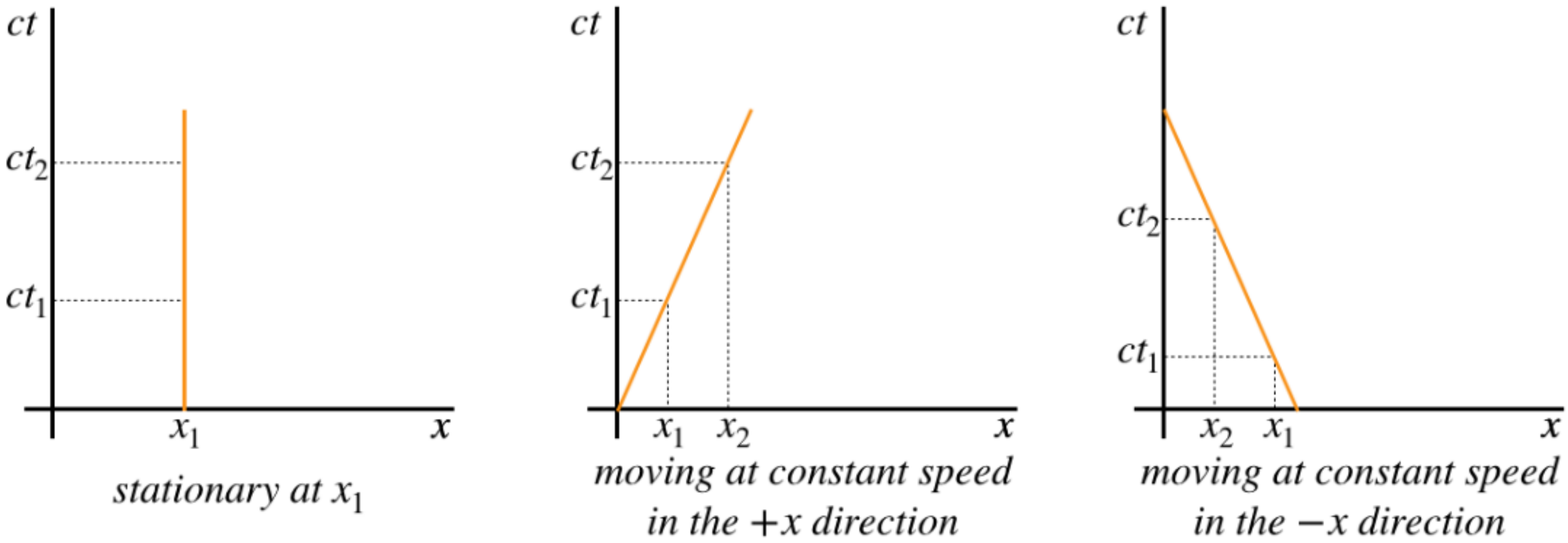
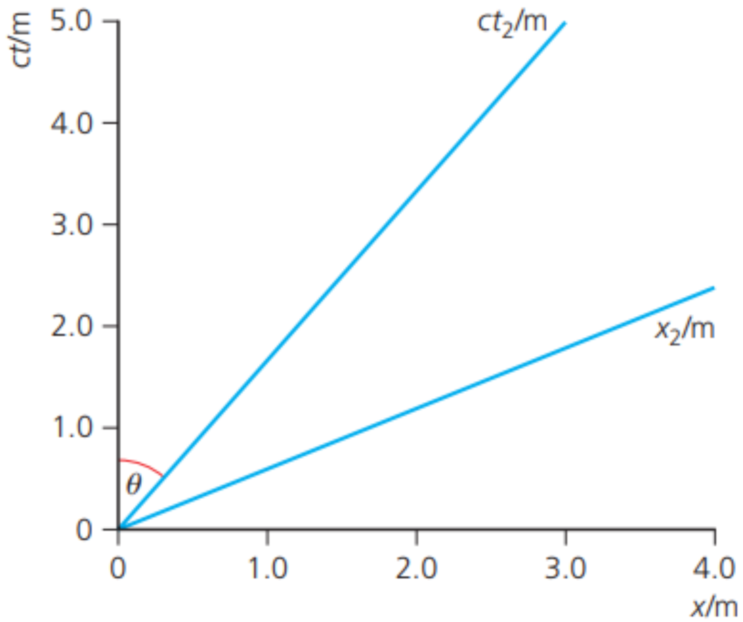


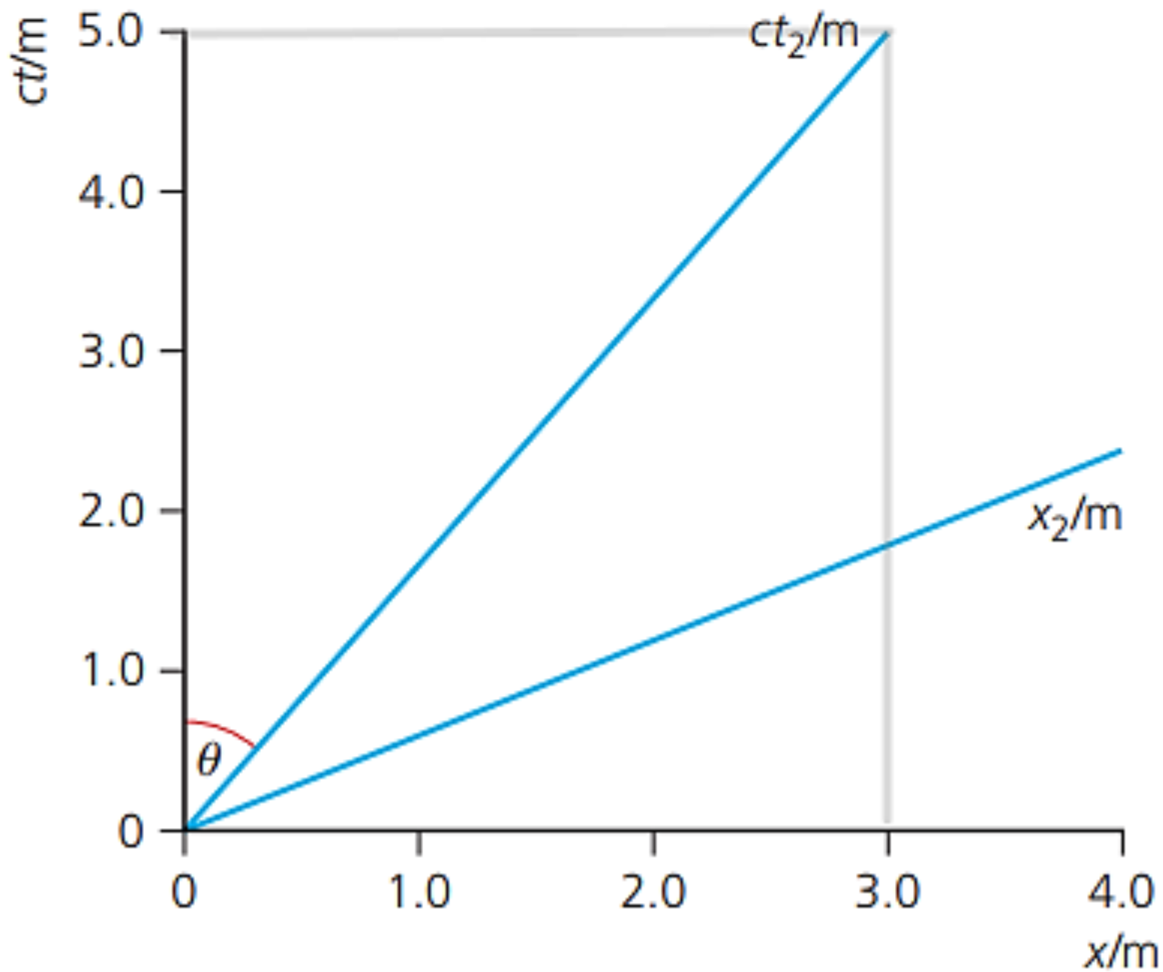
Figure A5.22 Space–time diagram with dashed blue lines representing separate world lines for different points that occur at the same time in reference frame S'

Simple worldlines



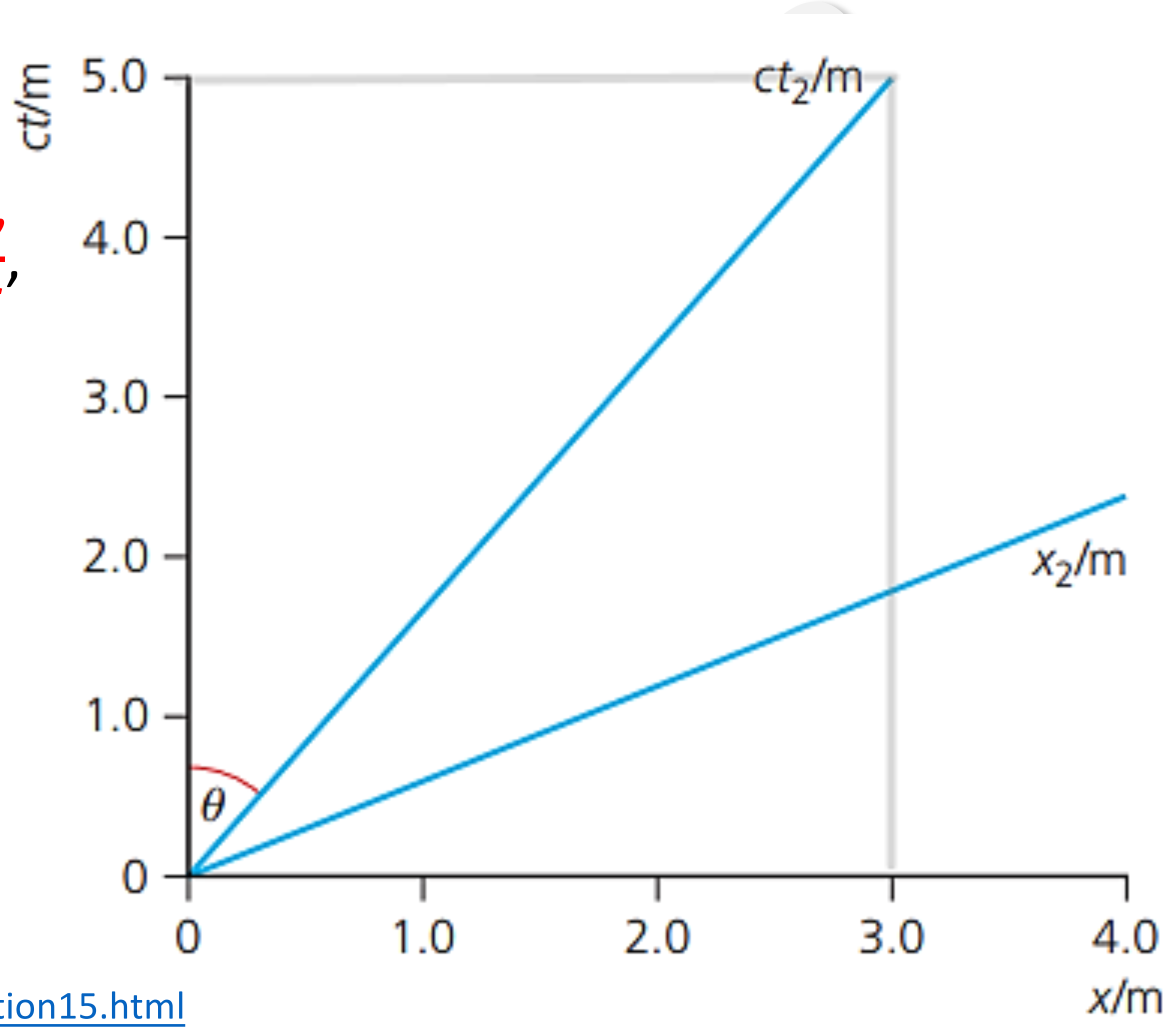
Use Figure A5.23 to determine the speed of the reference frame shown in blue relative to the other reference frame.





- $\tan\theta = \frac{v}{c} = \frac{3}{5} = 0.6$
- $v = 0.6\, c$

- $\tan\theta = \frac{v}{c} = \frac{3}{5} = 0.6$
- $v = 0.6\, c$
- Since $\boldsymbol{\gamma} = \frac{\mathbf{1}}{\sqrt{\left(\mathbf{1}-\left(\frac{v}{c}\right)^{2}\right)}}$, and $\tan\theta = \frac{v}{c}$,
- $\gamma = \frac{1}{\sqrt{1-(tan\ \theta)^2}}$



http://www.trell.org/div/minkowski.html

https://www1.phys.vt.edu/~takeuchi/relativity/notes/section15.html

STD for the invariant space time interval

- The space-time interval is invariant in Einstein's relativity.
- $(c\Delta t)^2 - (\Delta x)^2 = (c\,\Delta t')^2 - (\Delta x')^2$
- Quantity $\boldsymbol{(ct)^2 - (\Delta x)^2}$ is the space-time ***interval***

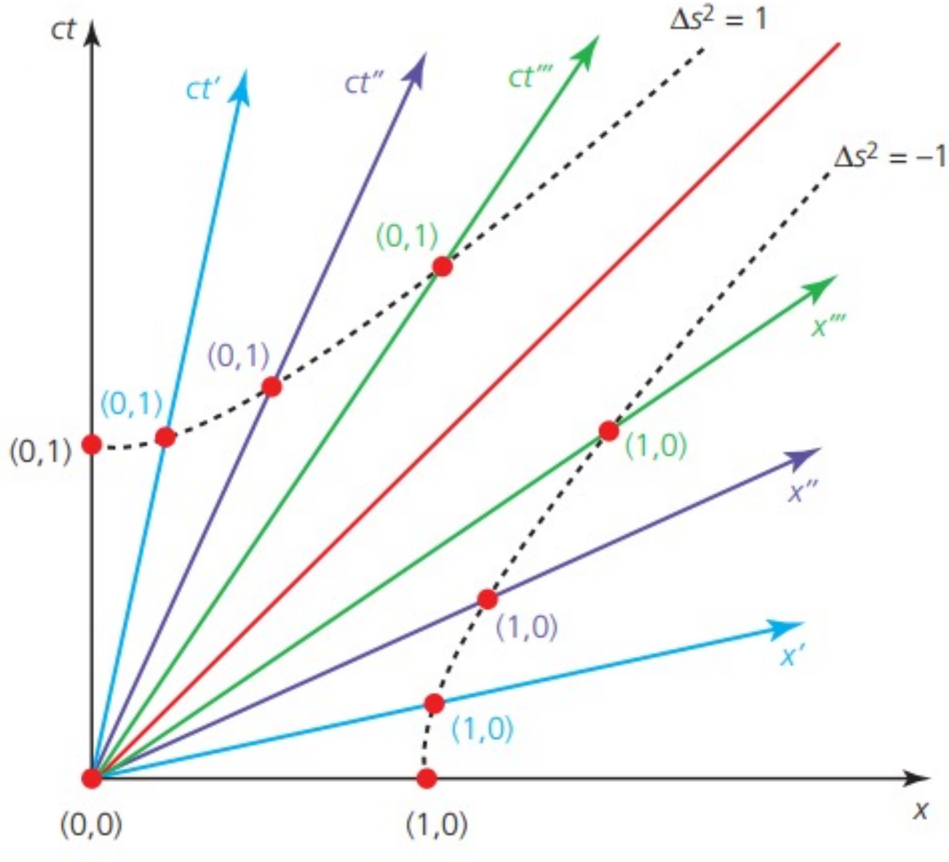


Figure A5.26 Two lines of constant space–time interval

43 a Use a ruler, calculator and Figure A5.27 to complete Table A5.1.

Table A5.1

Event	Coordinates in S (x, ct)	Coordinates in S' (x', ct')
A	(0, 0)	(0, 0)
B		(0.4, 0.4)
C	(1.6, 1.1)	(1, 0)
D		
E		(0, 1)
F		
G		

b List the order in which the events occur according to observers in both reference frame S and reference frame S'.

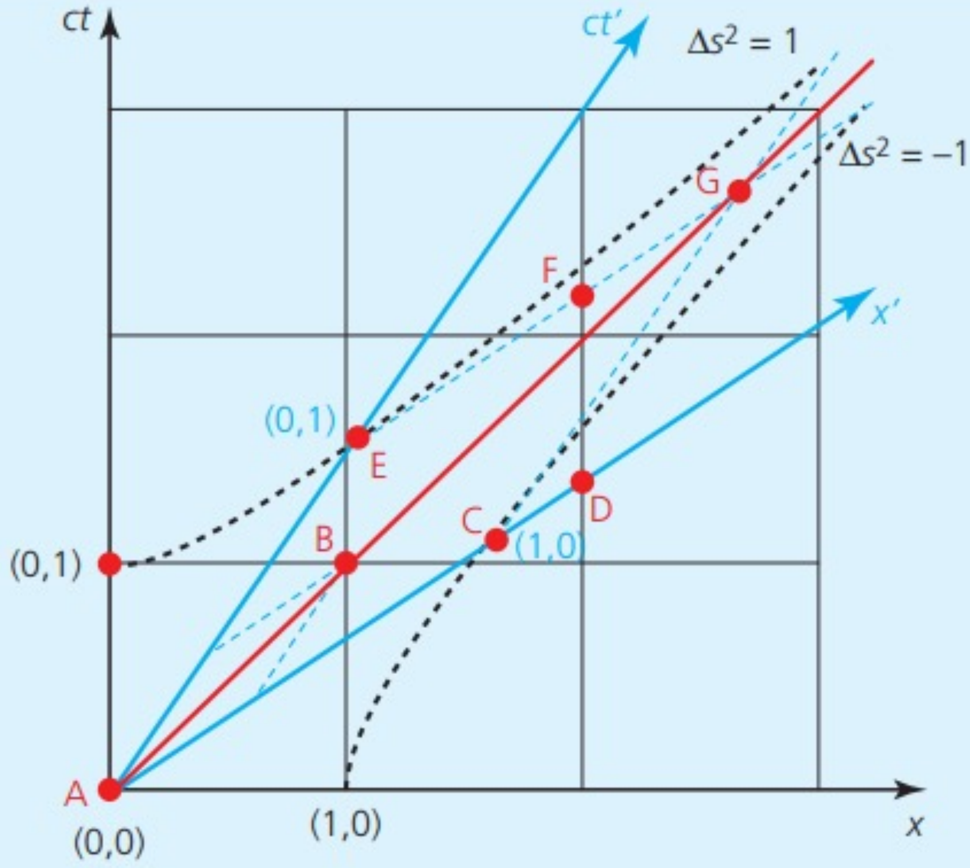
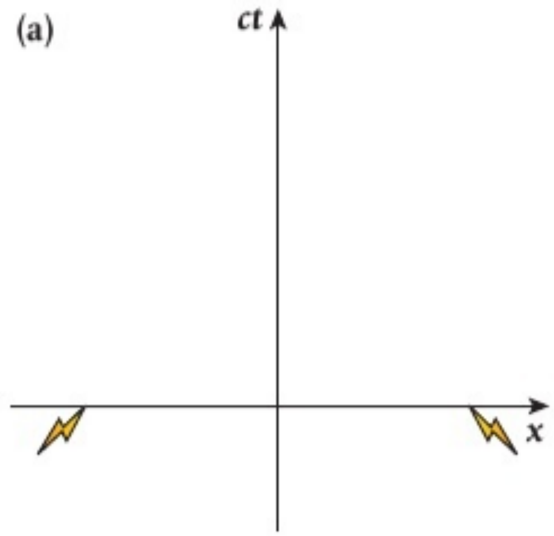


Figure A5.27 Space–time diagram showing seven events, labelled A to G, from two different reference frames

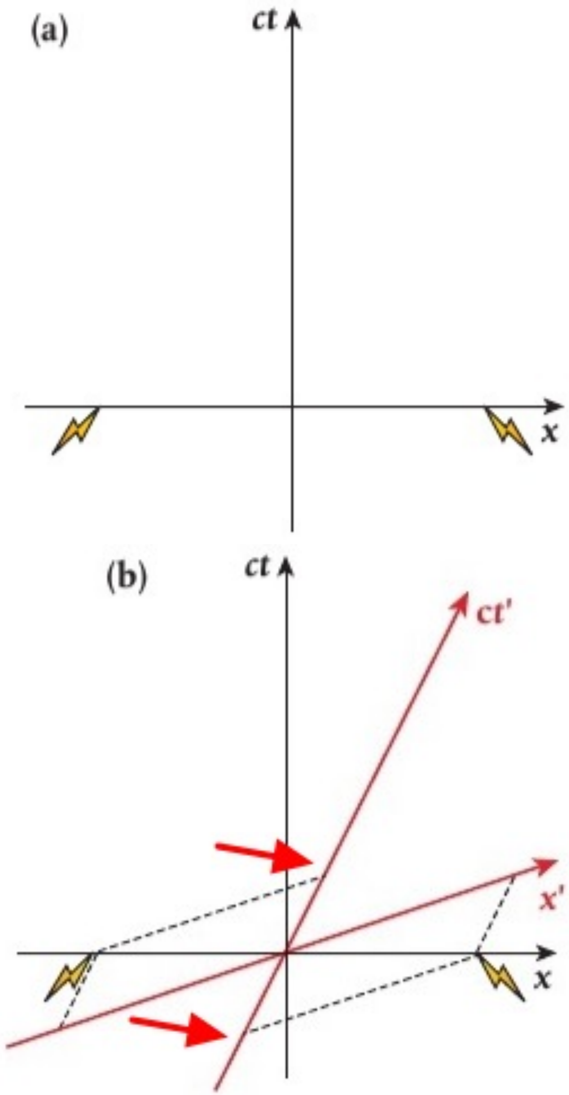
Simultaneity: STD

- Two events (lightning) occur simultaneously separated in space (left & right), but at same time



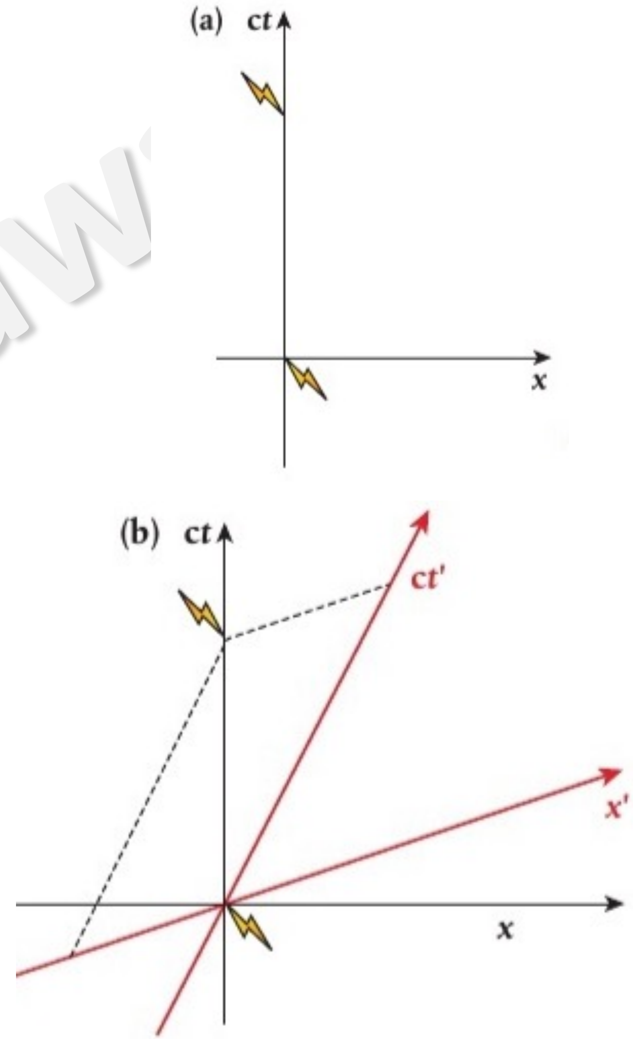
Simultaneity: STD

- Two events (lightning) occur simultaneously separated in space (left & right), but at same time
- When transformed to S' axes (red ones), the right event occurs before time, and the left one afterwards according to the moving FOR S', since S' is moving 'towards' the right



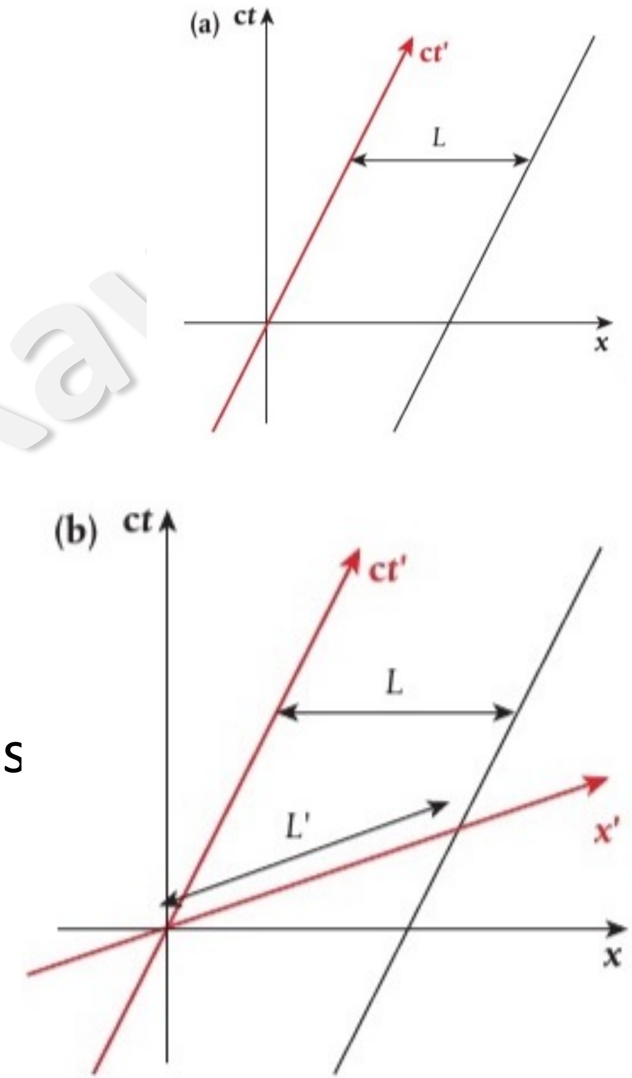
Time Dilation: STD

- In the bottom diagram, time (red ct' axis) is longer compared to time in S



Length Contraction: STD

- From the bottom diagram, time (red ct' axis) is longer compared to time in S
- Length L measured from S (simultaneous measurement in S
- Length gets shrunk
- Proper length L' is larger, whereas it becomes smaller (L) When measured from the fixed frame S



The Twin Paradox: Statement of the problem

- Twin B travels away from twin A at $0.661\ c$ and then returns
- Both see the other twin's clock ticking slowly (aging less)
- Both think they will be older when B comes back
- But both cannot be older than each other, hence paradox
- Actually, travelling twin B will be younger

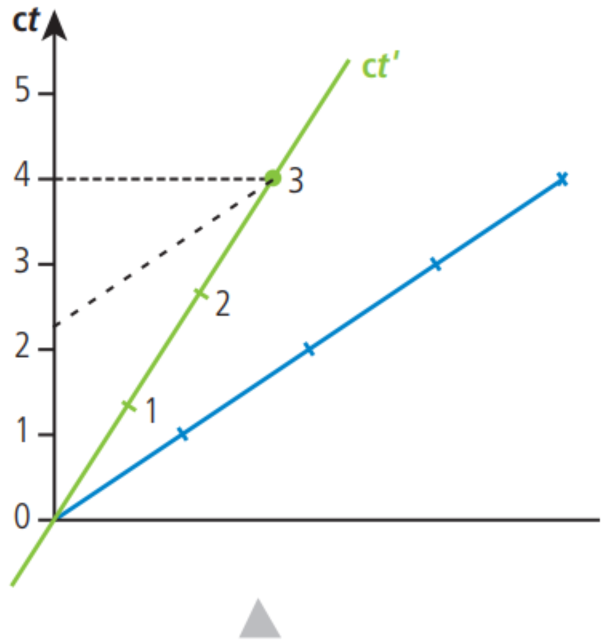
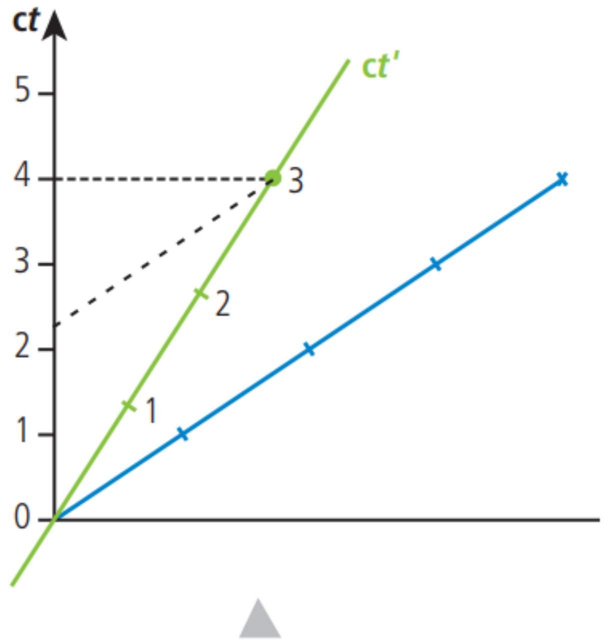


Figure 9.47 The first part of the twin's journey.

The Twin Paradox

- B's FOR is ct' so she moves along the green ($x' = 0$) line in space-time
- After 3 years (in FOR ct') twin B decides to return, which will be 4 years in A's FOR (black ct line)



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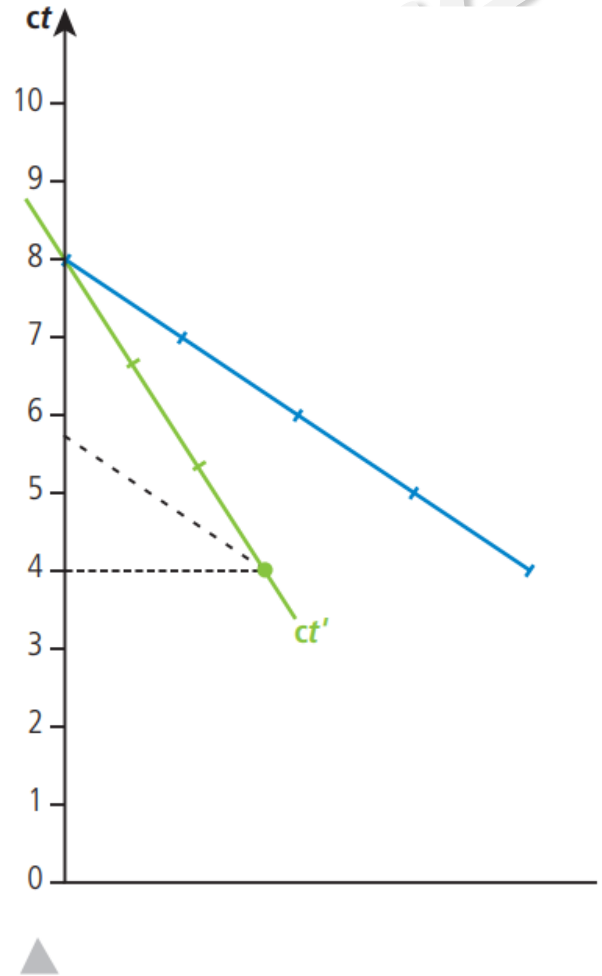


Figure 9.48 The second part of the journey.

The Twin Paradox

- B's FOR is ct' so she moves along the green $(x' = 0)$ line in space-time
- After 3 years (in FOR ct') twin B decides to return, which will be 4 years in A's FOR (black ct line)
- Upon return, B is travelling in opposite direction, hence the STD has her worldline (green) tilted the other way
- Hence what was 6 years for B, was 8 years for A

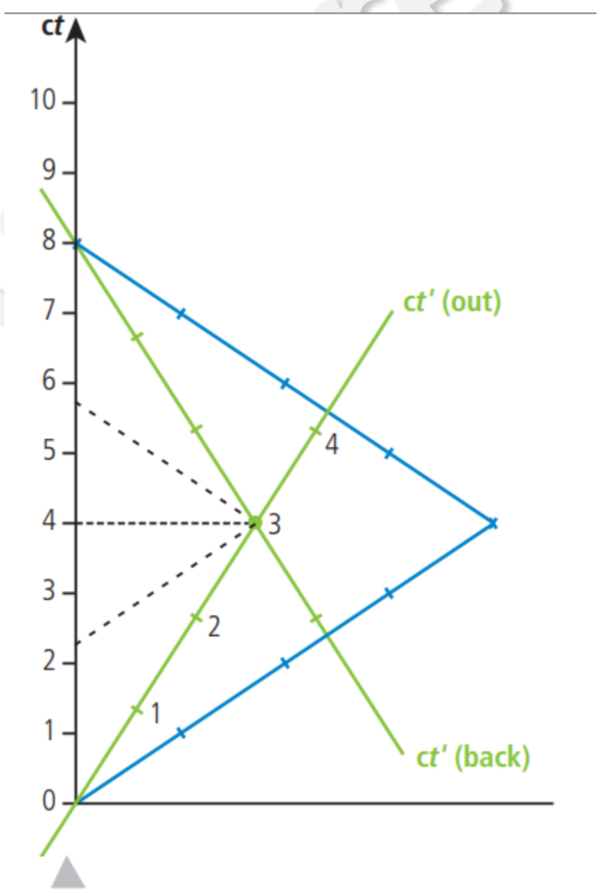
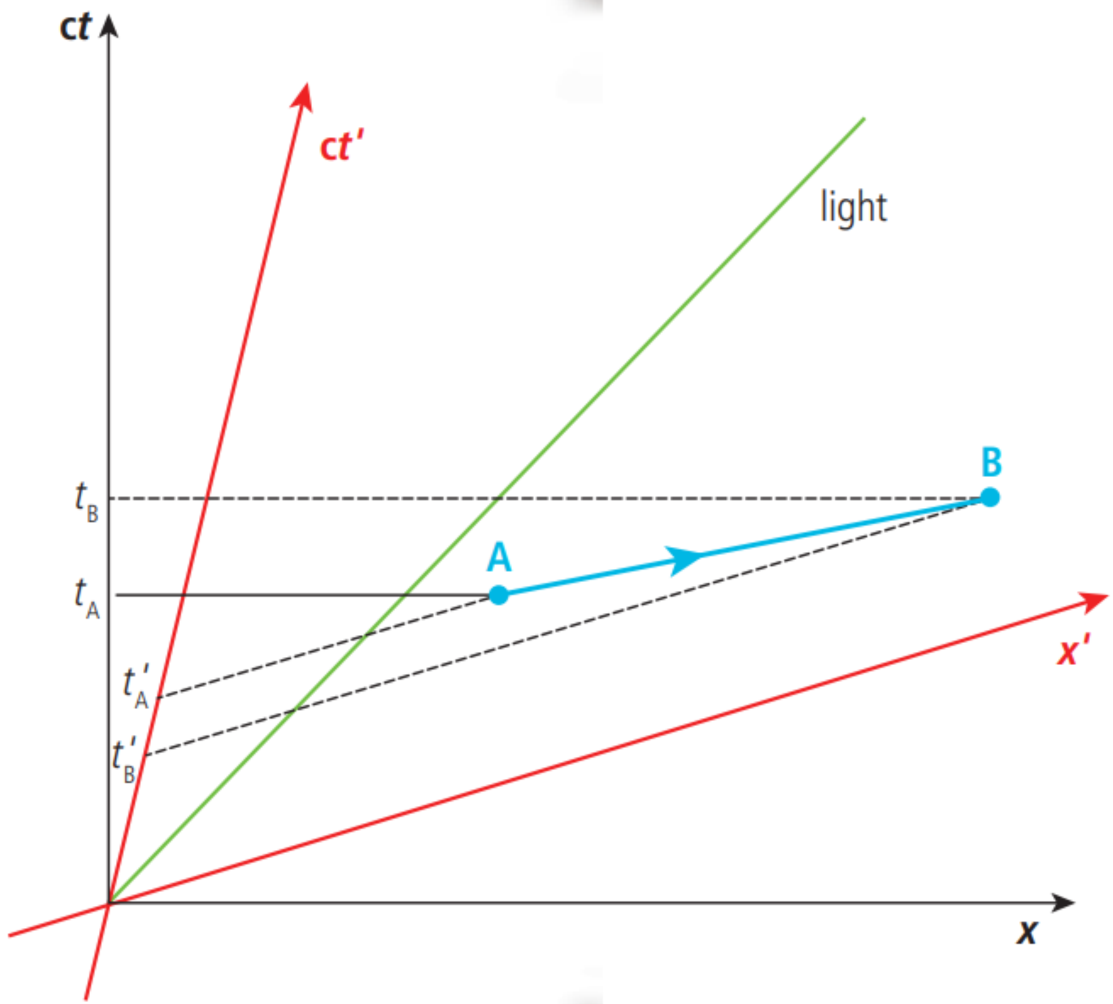


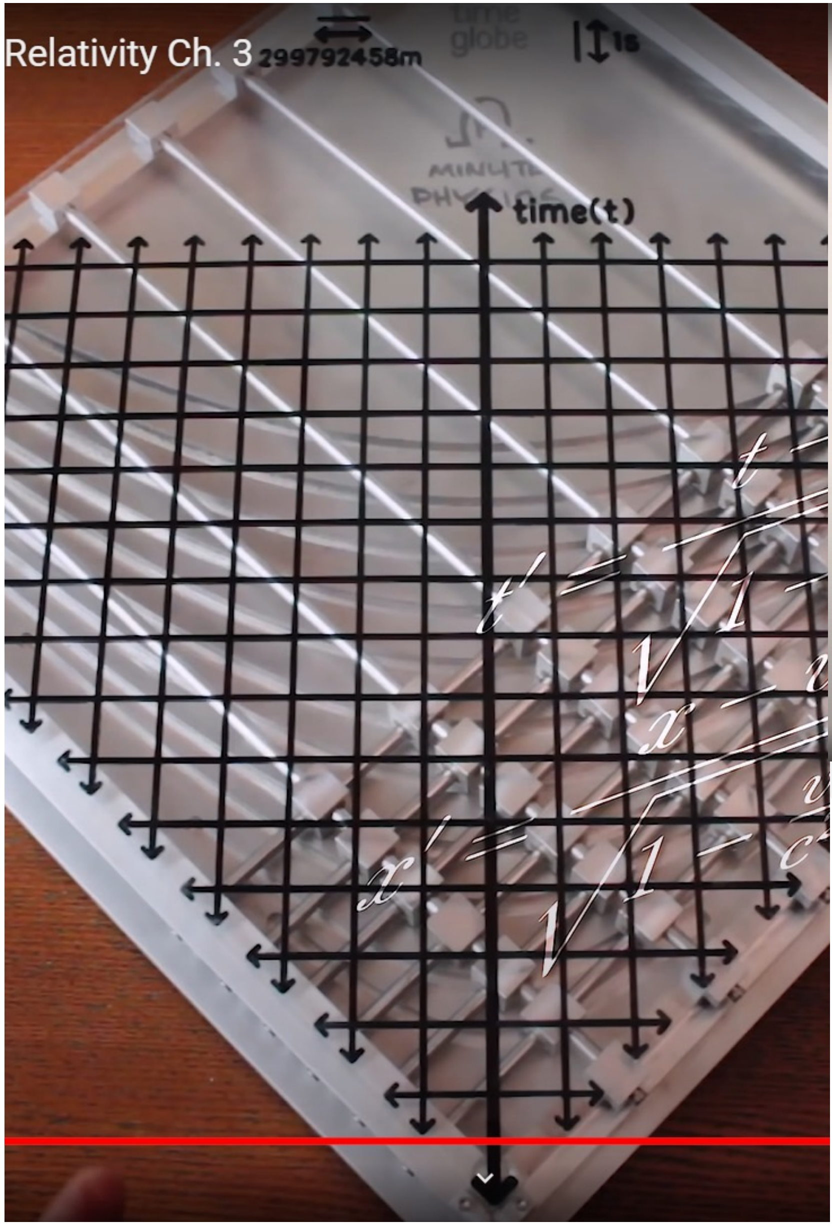
Figure 9.49 The complete journey

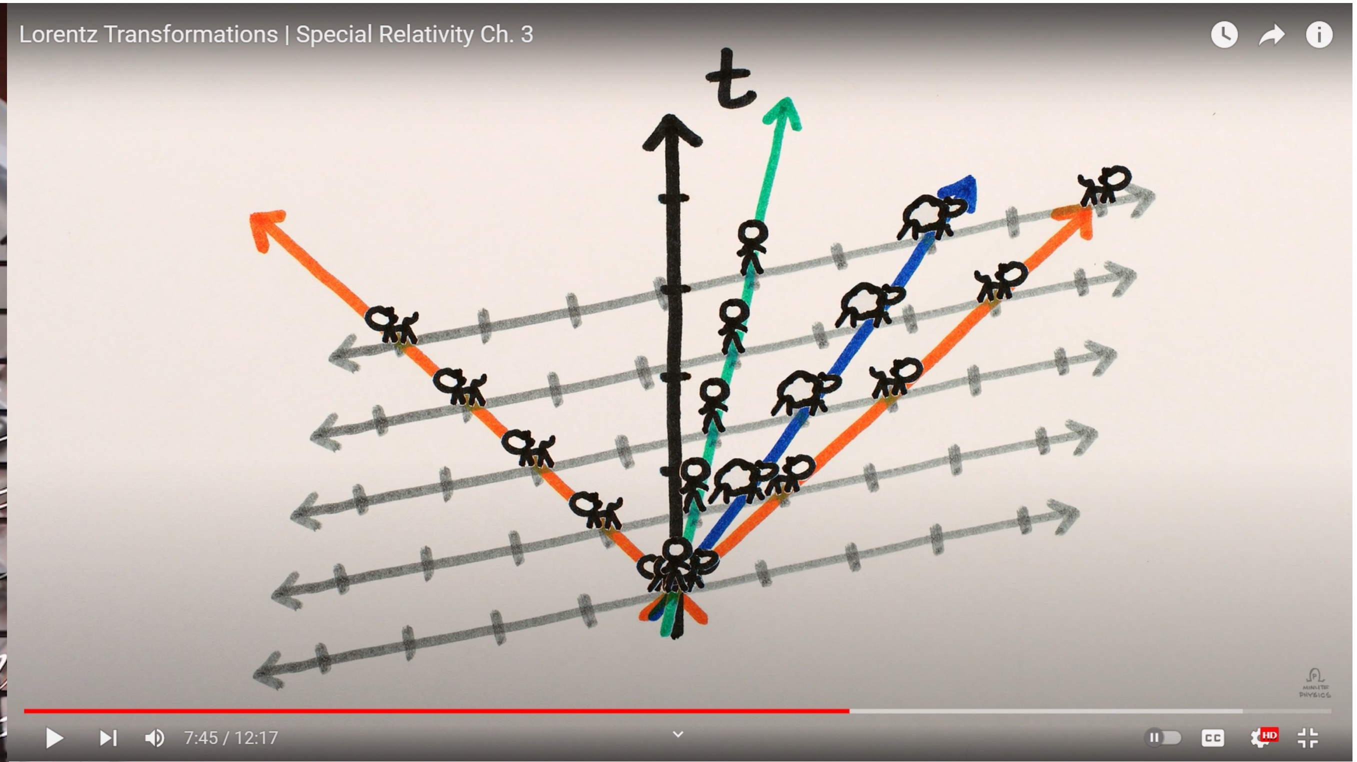
Greater than c: time travel

- If we start at time t_A and end at time t_B
- As observed from S' t_B occurs at 'before' t_B which is not possible (reaching before starting)



- A spacecraft is moving away from earth at speed $0.60\,c$. At $ct = 0$, just as the spacecraft begins to move, a flash of light leaves a space station which is at rest $4.0\,ly$ from earth, according to earth. Using a spacetime diagram, estimate the time the flash of light arrives at the spacecraft according to earth and the spacecraft.
- $\tan\theta = 0.6 = \frac{v}{c} \therefore \theta = \tan^{-1} 0.6 = 30.96\,°$





https://www.youtube.com/watch?v=Rh0pYtQG5wI